

# MAZĀ MATEMĀTIKAS UNIVERSITĀTE



LATVIJAS  
UNIVERSITĀTE

ANNO 1919



LATVIJAS UNIVERSITĀTE  
Fizikas un Matemātikas fakultāte  
A. Liģas Nolikuma matemātikas skola

FIZMATI.LV



# Algoritmi

## un to sarežģītība

asociētais profesors Jānis Buls

lektors Edmunds Cers

un uzticamie doktoranti

Raivis Bēts, Inese Bērziņa, Līga Kuleša

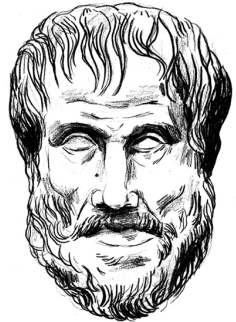
# Plāns

Šodien centīsimies noskaidrot:

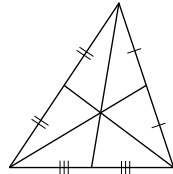
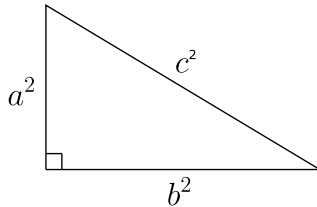
- 1 Kas ir algoritms
- 2 Kas ir Tjūringa mašīna
- 3 Ko nozīmē P un NP

# Matemātika

- Vārds “matemātika” cēlies no grieķu *μαθημα*, kas nozīmē mācīties, studēt
- Aristotelis matemātiku definēja kā zinātni par daudzumiem
- Mūsdienās vairs nemākam matemātiku definēt !

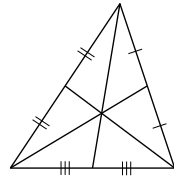
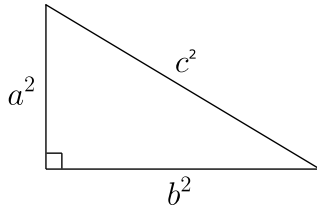


- Ģeometrijā:



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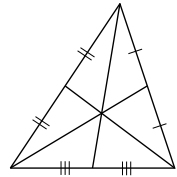
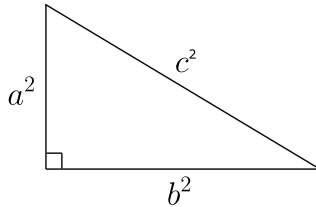


- Tue-Morsa vārds:

a

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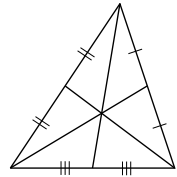
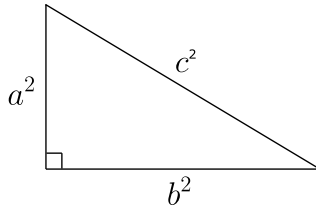


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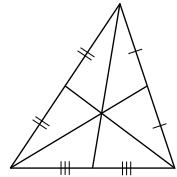
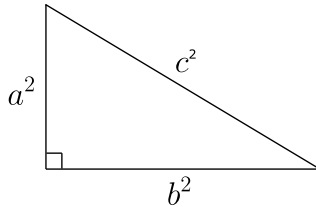
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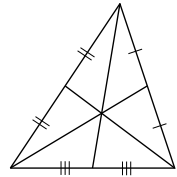
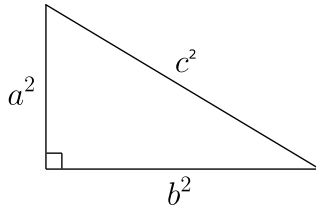


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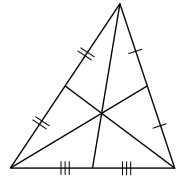
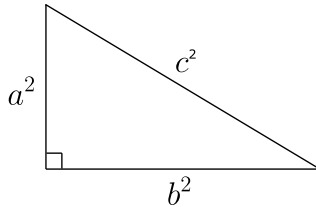


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abbabaabbaabba

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# Matemātika



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# Algoritmi

Dažreiz nepietiek ar formulām — kā bez kalkulatora saskaitīt

$$3881 + 2753 = ?$$

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Varam saskaitīt stabiņā:

$$\begin{array}{r} \phantom{+} \phantom{3^1} \phantom{8^1} \phantom{8} \phantom{1} \\ + \phantom{3^1} \phantom{8^1} \phantom{8} \phantom{1} \\ \hline \phantom{3^1} \phantom{8^1} \phantom{8} \phantom{1} \\ 6 \phantom{0} \phantom{0} \phantom{0} \phantom{0} \end{array}$$

# Algoritmi

$$x^3 + 4x^2 - 5x = 0$$

⇓

$$x(x^2 + 4x - 5) = 0$$

↙

↘

$$(x^2 + 4x - 5) = 0$$

$$x = 0$$

↙

$$x = 1$$

↘

$$x = -5$$

# Algoritmi

- Algoritms ir soļu uzskaitījums, kas jāveic, lai sasniegtu vēlamu rezultātu.
- Tā nosaukts par godu izcilajam Persiešu matemātiķim **Abū Abdullam Muhamedam ibn-Musam al-Harizmi** (apm. 780–850 g.)





## Sarežģītāki algoritmi

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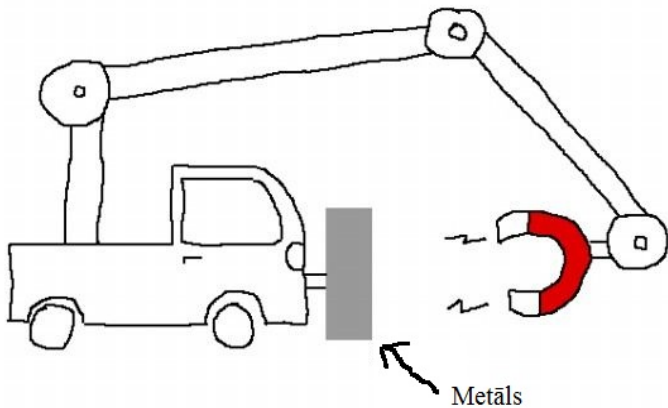
## Hilberta problēma

- Leibnics sapņoja uzbūvēt mašīnu, kas manipulētu matemātiskus simbolus un pierādītu izteiksmes.
- Mūsdienu valodā mēs jautātu “Vai iespējams atrast algoritmu, kas pasaka, vai matemātiska izteiksme ir patiesa?”
- Šo jautājumu formulējis **Dāvids Hilberts** (1862.–1943.) tālajā 1928. gadā.

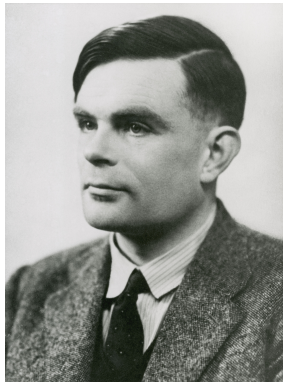


# Neiespējamība

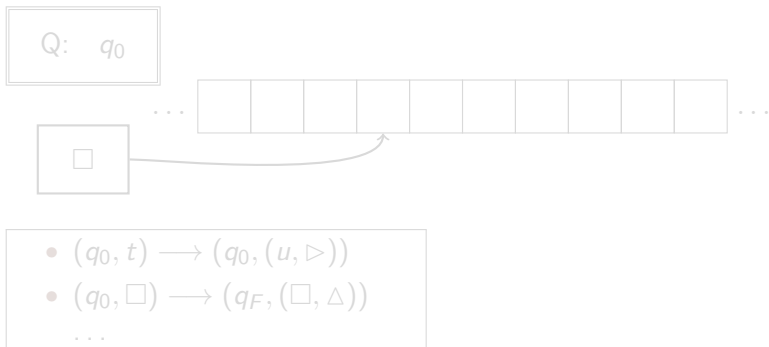
## Mūžīgais dzinējs



# Alonzo Čērčs un Alans Tjūrings

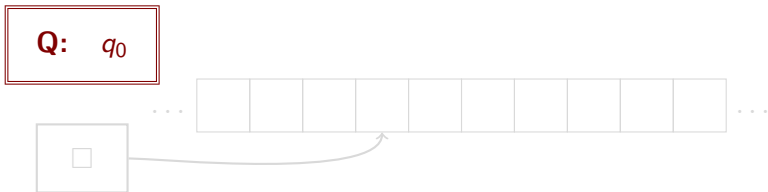


# Tjūringa mašīna



- Vadības bloks, ar galīgu skaitu stāvokļu.
- Lente (vai lentes), datu ievadei, apstrādei un izvadei
- Galva, kas norāda uz pozīciju uz lentas
- Likumu saraksts

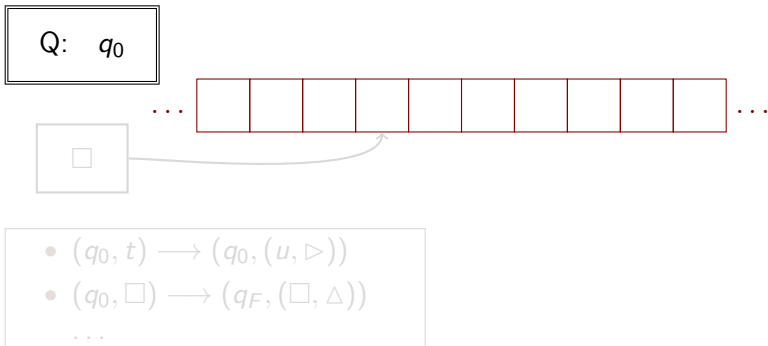
# Tjūringa mašīna



- $(q_0, t) \rightarrow (q_0, (u, \triangleright))$
- $(q_0, \square) \rightarrow (q_F, (\square, \Delta))$
- $\dots$

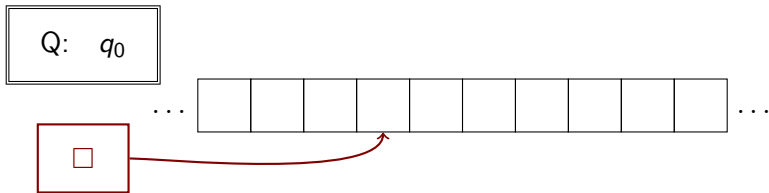
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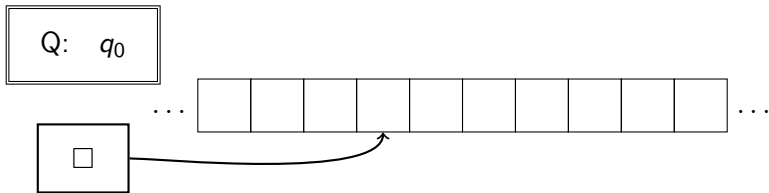
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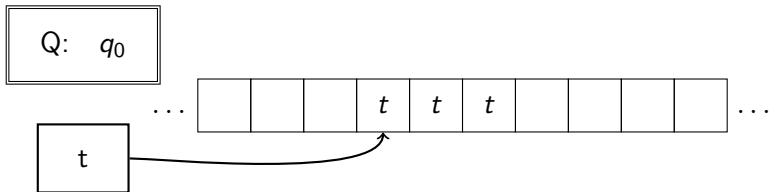


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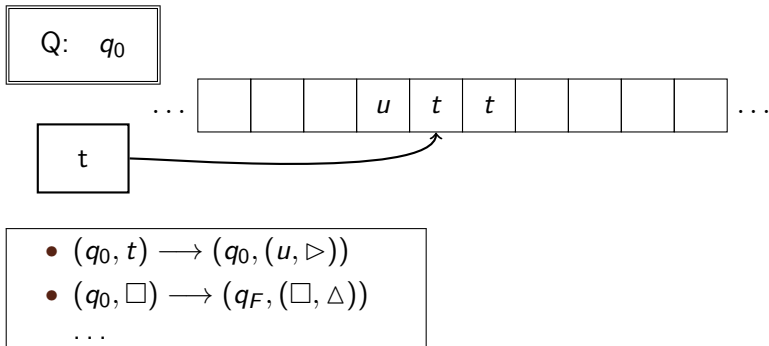
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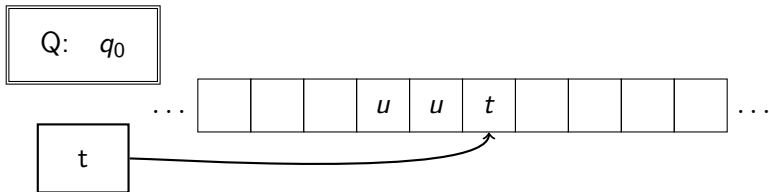
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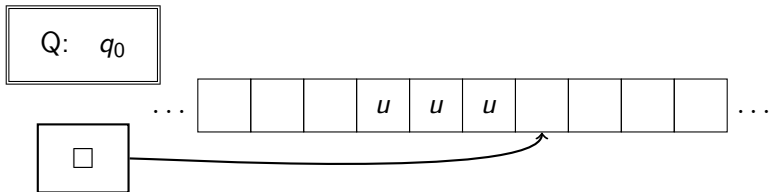
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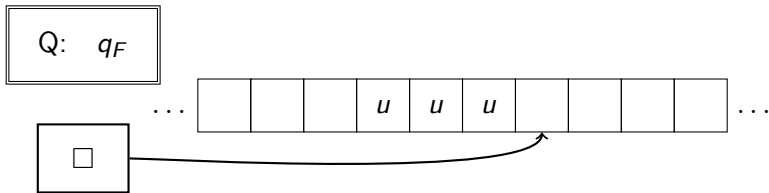
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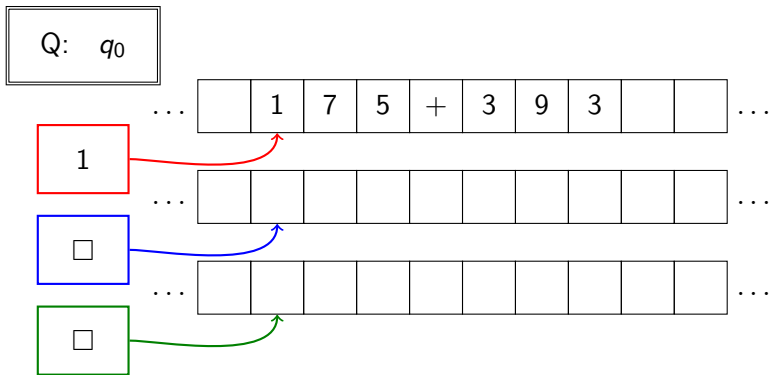
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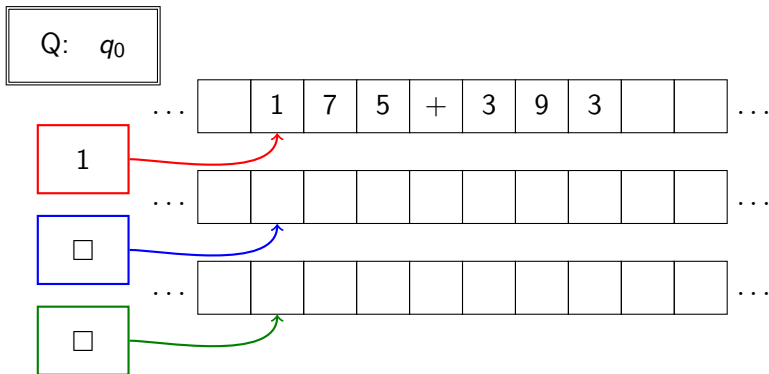
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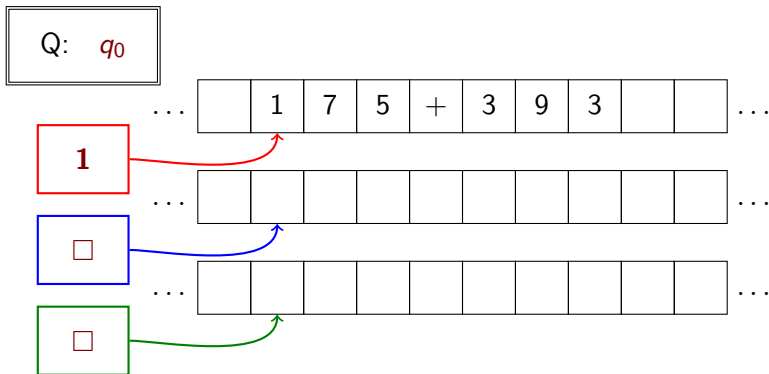
Likums:

# Tjūringa mašīna



Likums:  $(q_0, n, \square) \rightarrow (q_0, (n, \triangleright), (\square, \Delta), (\square, \Delta))$

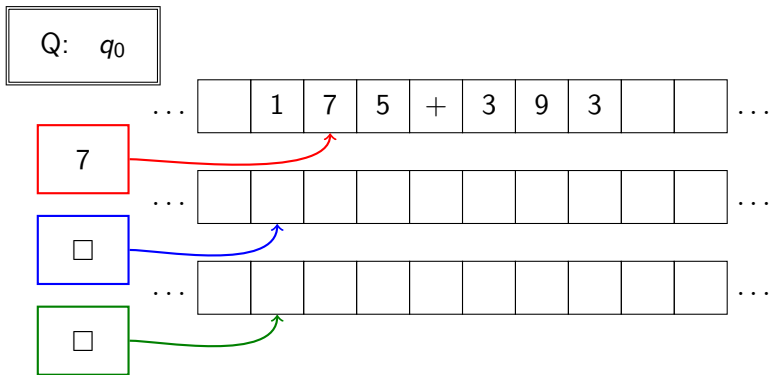
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Likums:  $(q_0, n, \square) \rightarrow (q_0, (n, \triangleright), (\square, \Delta), (\square, \Delta))$

## Tjūringa mašīna

$(q_0, n, \square) \longrightarrow (q_0, (n, \triangleright), (\square, \Delta), (\square, \Delta))$  patiesībā **10** likumi:

$$1 \quad (q_0, 0, \square) \longrightarrow (q_0, (0, \triangleright), (\square, \Delta), (\square, \Delta))$$

$$2 \quad (q_0, 1, \square) \longrightarrow (q_0, (1, \triangleright), (\square, \Delta), (\square, \Delta))$$

$$3 \quad (q_0, 2, \square) \longrightarrow (q_0, (2, \triangleright), (\square, \Delta), (\square, \Delta))$$

$$4 \quad (q_0, 3, \square) \longrightarrow (q_0, (3, \triangleright), (\square, \Delta), (\square, \Delta))$$

$$5 \quad (q_0, 4, \square) \longrightarrow (q_0, (4, \triangleright), (\square, \Delta), (\square, \Delta))$$

$$6 \quad (q_0, 5, \square) \longrightarrow (q_0, (5, \triangleright), (\square, \Delta), (\square, \Delta))$$

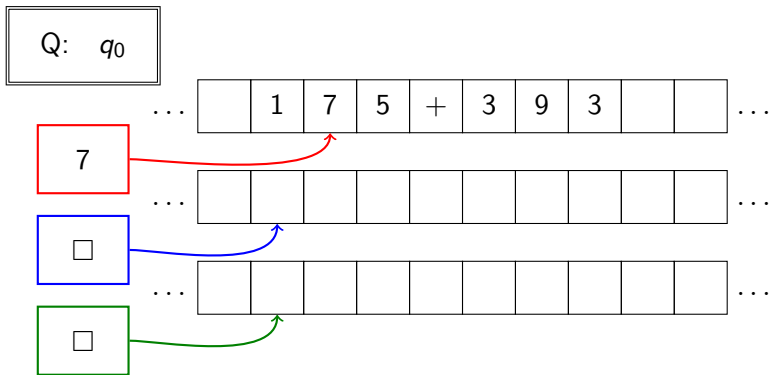
$$7 \quad (q_0, 6, \square) \longrightarrow (q_0, (6, \triangleright), (\square, \Delta), (\square, \Delta))$$

$$8 \quad (q_0, 7, \square) \longrightarrow (q_0, (7, \triangleright), (\square, \Delta), (\square, \Delta))$$

$$9 \quad (q_0, 8, \square) \longrightarrow (q_0, (8, \triangleright), (\square, \Delta), (\square, \Delta))$$

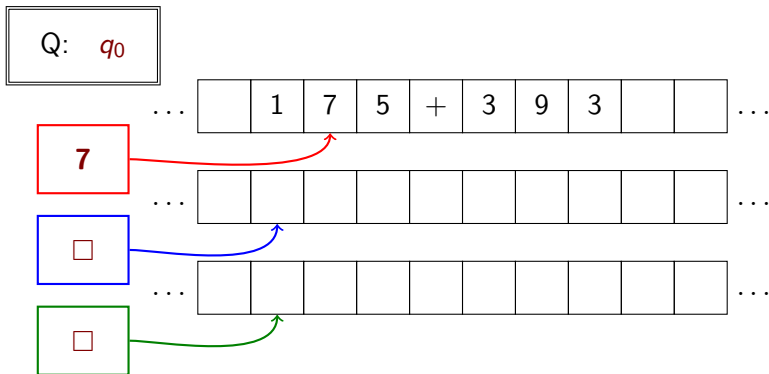
$$10 \quad (q_0, 9, \square) \longrightarrow (q_0, (9, \triangleright), (\square, \Delta), (\square, \Delta))$$

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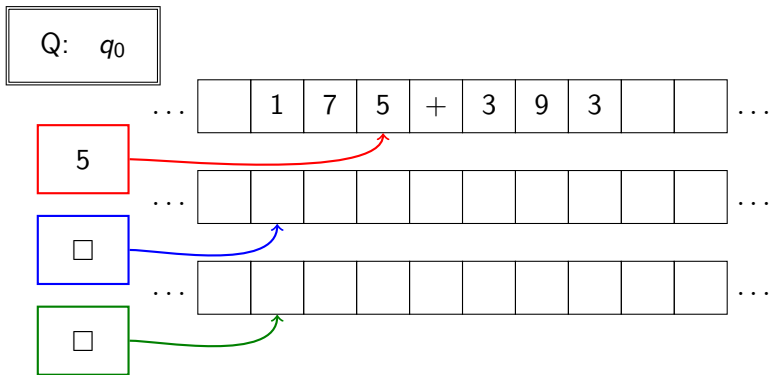
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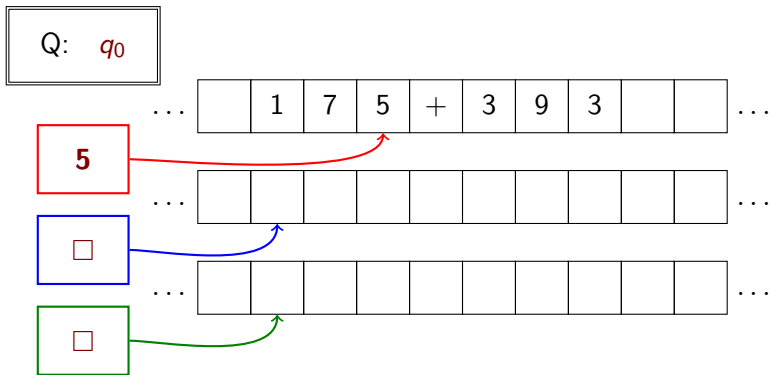
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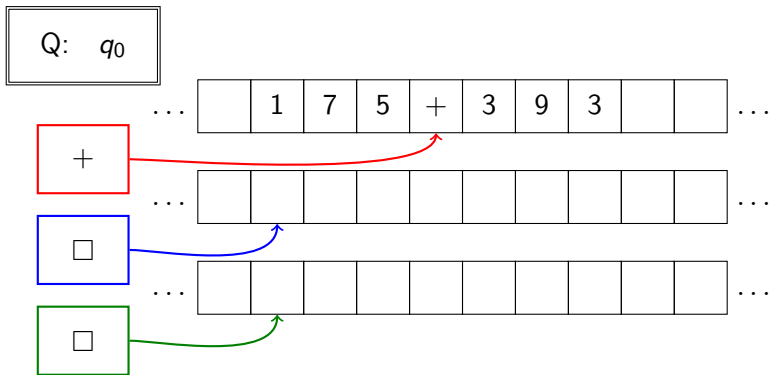
Likums:  $(q_0, n, \square) \rightarrow (q_0, (n, \triangleright), (\square, \Delta), (\square, \Delta))$

# Tjūringa mašīna



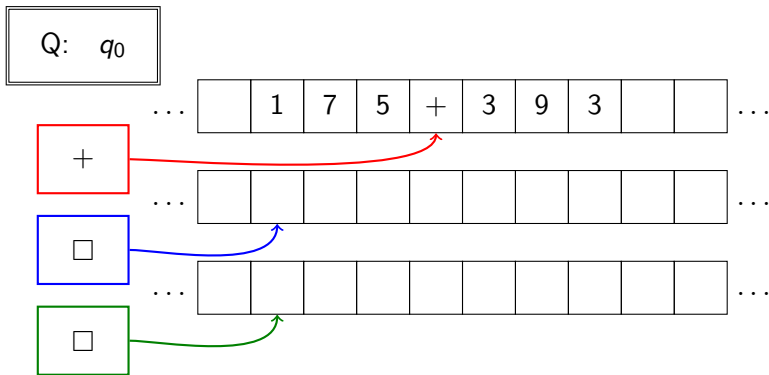
Likums:  $(q_0, n, \square) \rightarrow (q_0, (n, \triangleright), (\square, \Delta), (\square, \Delta))$

# Tjūringa mašīna



Likums:  $(q_0, n, \square) \rightarrow (q_0, (n, \triangleright), (\square, \Delta), (\square, \Delta))$

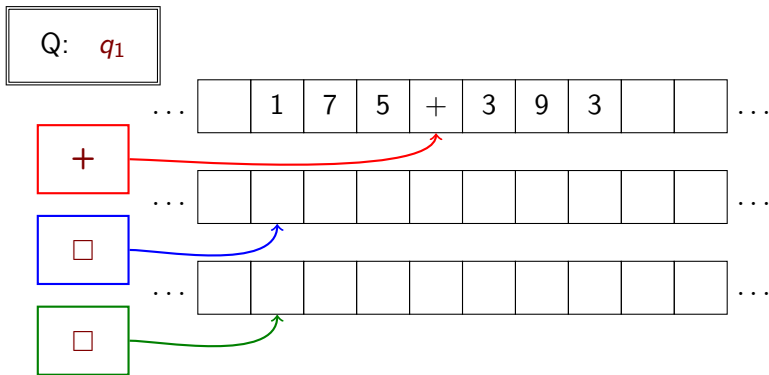
# Tjūringa mašīna



Likums:  $(q_0, +, \square) \rightarrow (q_1, (+, \triangleright), (\square, \Delta), (\square, \Delta))$

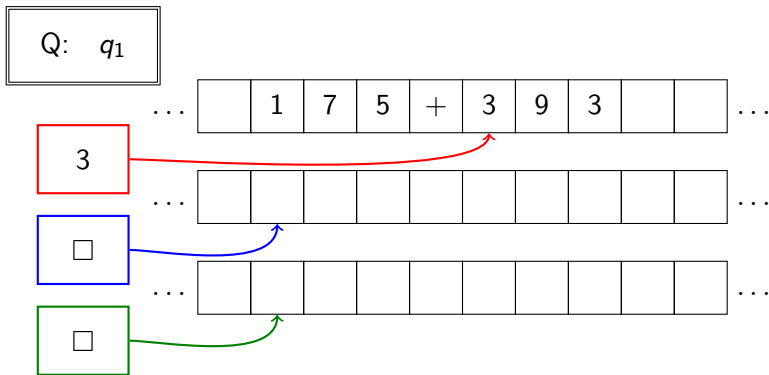


# Tjūringa mašīna



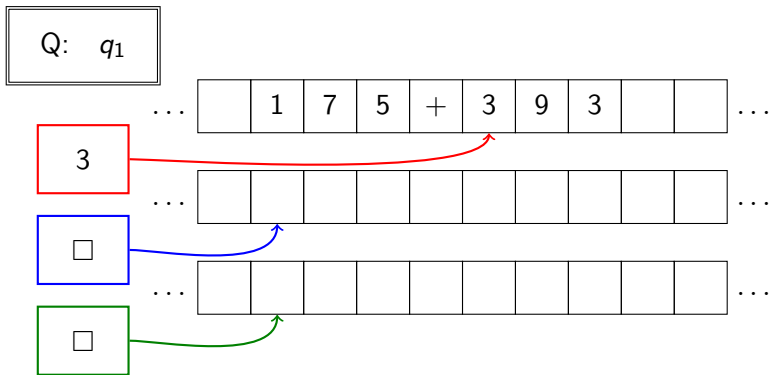
Likums:  $(q_0, +, \square) \rightarrow (q_1, (+, \triangleright), (\square, \Delta), (\square, \Delta))$

# Tjūringa mašīna



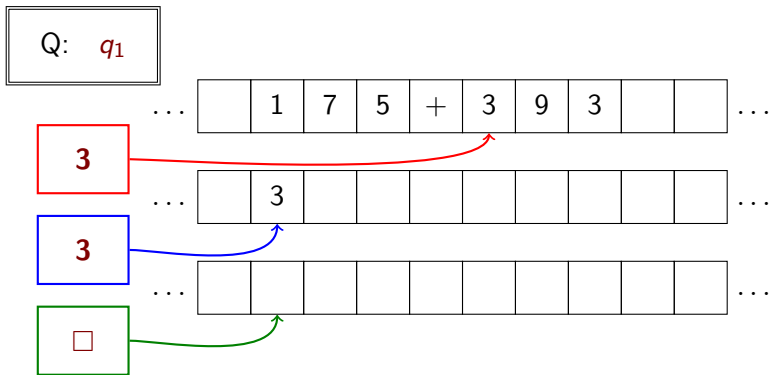
Likums:  $(q_0, +, \square) \rightarrow (q_1, (+, \triangleright), (\square, \Delta), (\square, \Delta))$

# Tjūringa mašīna



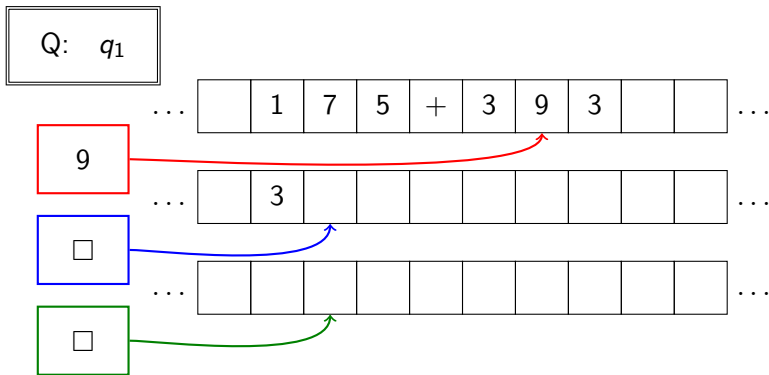
Likums:  $(q_1, n, \square) \rightarrow (q_1, (n, \triangleright), (n, \triangleright), (\square, \triangleright))$

# Tjūringa mašīna



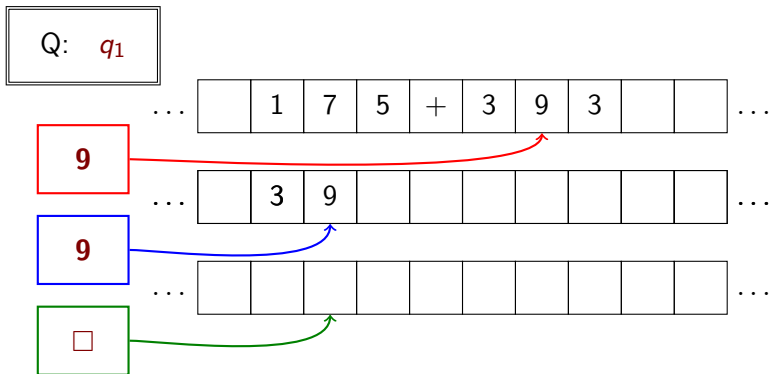
Likums:  $(q_1, n, \square) \rightarrow (q_1, (n, \triangleright), (n, \triangleright), (\square, \triangleright))$

# Tjūringa mašīna



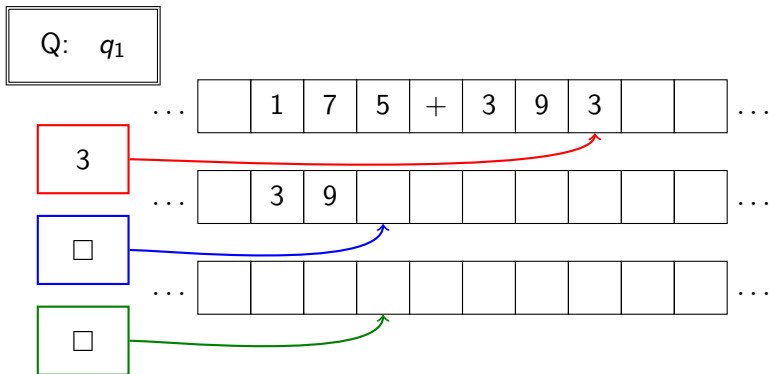
Likums:  $(q_1, n, \square) \rightarrow (q_1, (n, \triangleright), (n, \triangleright), (\square, \triangleright))$

# Tjūringa mašīna



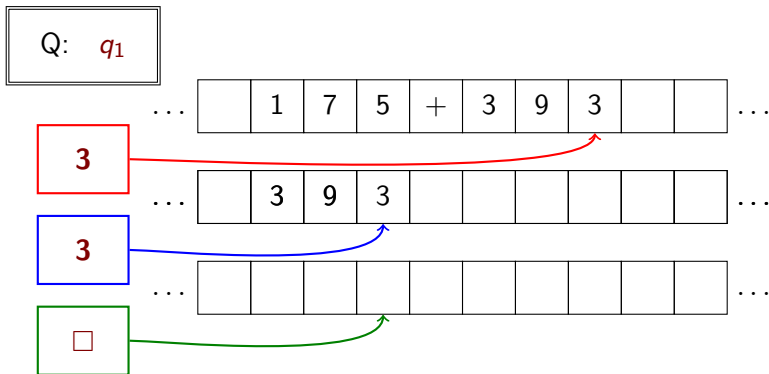
Likums:  $(q_1, n, \square) \rightarrow (q_1, (n, \triangleright), (n, \triangleright), (\square, \triangleright))$

# Tjūringa mašīna



Likums:  $(q_1, n, \square) \rightarrow (q_1, (n, \triangleright), (n, \triangleright), (\square, \triangleright))$

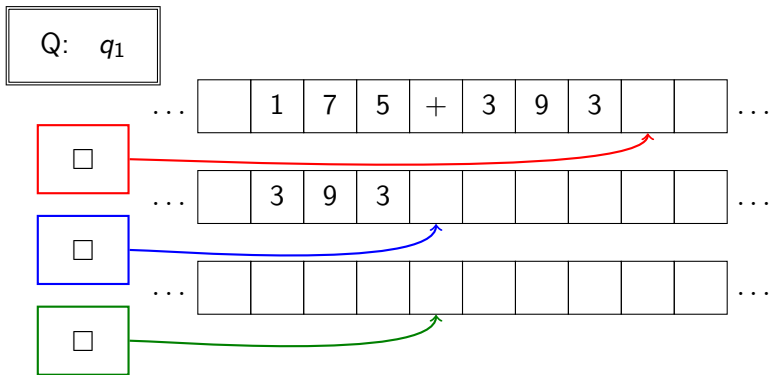
# Tjūringa mašīna



Likums:  $(q_1, n, \square) \rightarrow (q_1, (n, \triangleright), (n, \triangleright), (\square, \triangleright))$

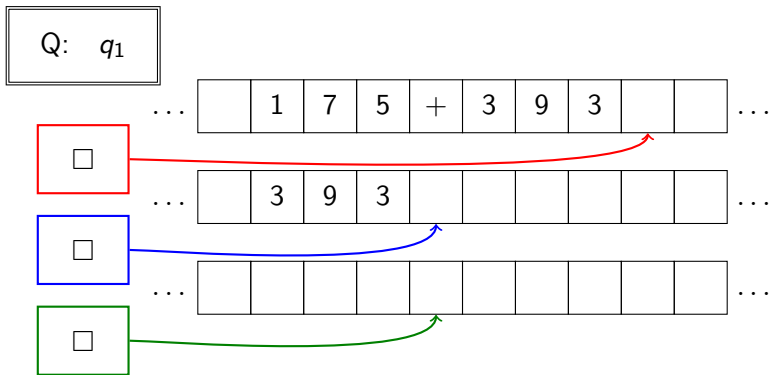


# Tjūringa mašina



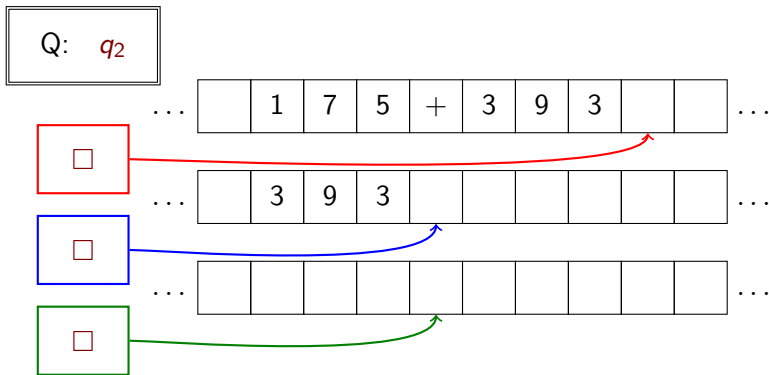
Likums:  $(q_1, n, \square) \rightarrow (q_1, (n, \triangleright), (n, \triangleright), (\square, \triangleright))$

# Tjūringa mašīna



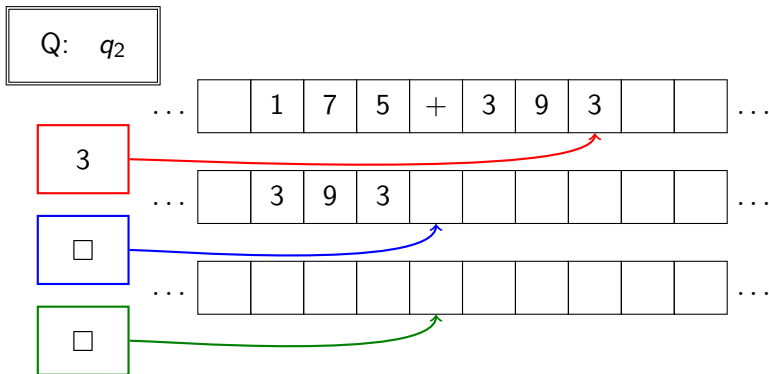
Likums:  $(q_1, \square, \square) \rightarrow (q_2, (\square, \triangleleft), (\square, \triangle), (\square, \triangle))$

# Tjūringa mašina



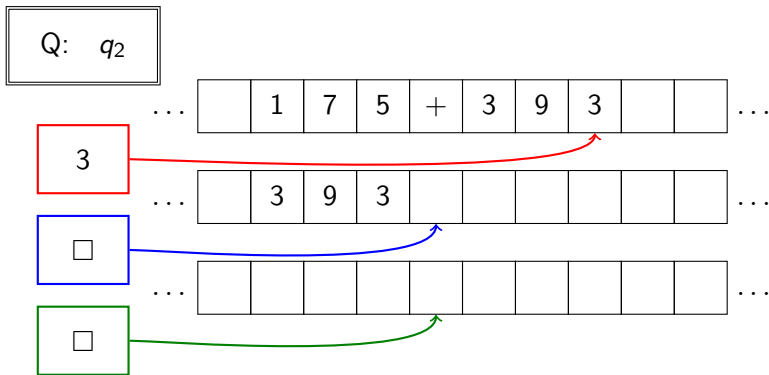
Likums:  $(q_1, \square, \square) \rightarrow (q_2, (\square, \triangleleft), (\square, \triangle), (\square, \triangle))$

# Tjūringa mašīna



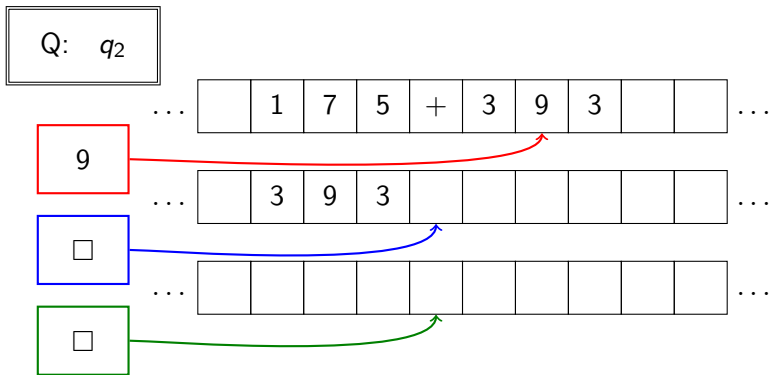
Likums:  $(q_1, \square, \square) \rightarrow (q_2, (\square, \triangleleft), (\square, \triangle), (\square, \triangle))$

# Tjūringa mašīna



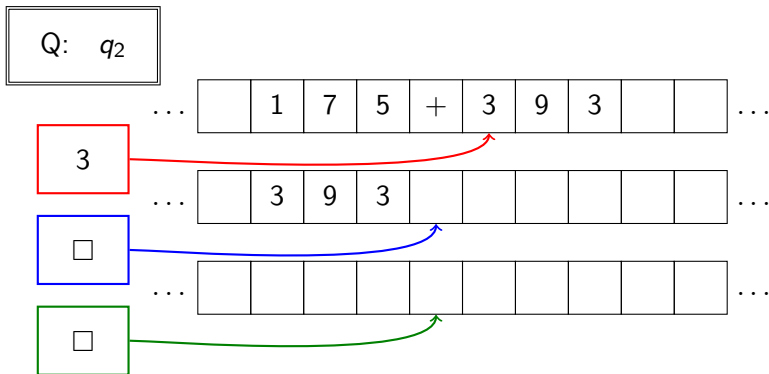
Likums:  $(q_2, n, \square) \rightarrow (q_2, (n, \triangleleft), (\square, \Delta), (\square, \Delta))$

# Tjūringa mašina



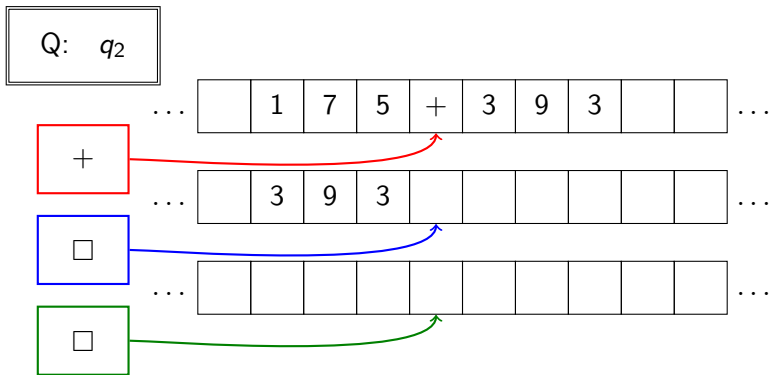
Likums:  $(q_2, n, \square) \rightarrow (q_2, (n, \triangleleft), (\square, \triangle), (\square, \triangle))$

# Tjūringa mašina



Likums:  $(q_2, n, \square) \rightarrow (q_2, (n, \triangleleft), (\square, \triangle), (\square, \triangle))$

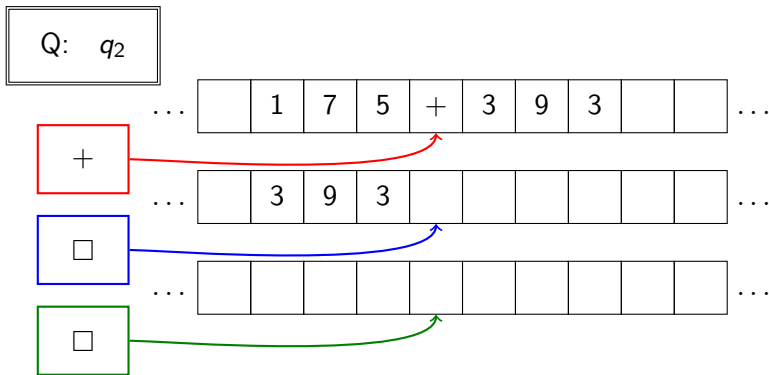
# Tjūringa mašīna



Likums:  $(q_2, n, \square) \rightarrow (q_2, (n, \triangleleft), (\square, \triangle), (\square, \triangle))$

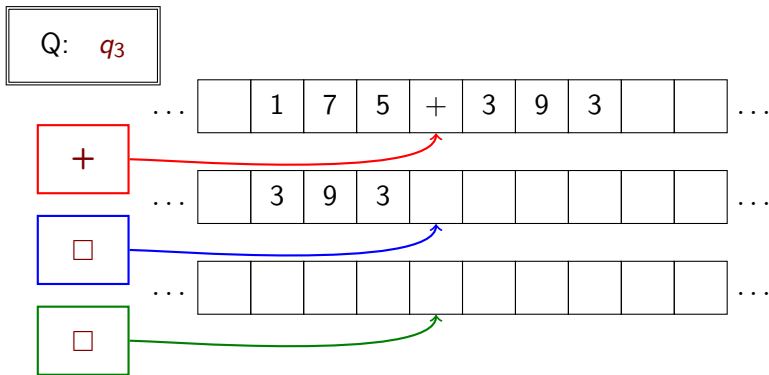


# Tjūringa mašina



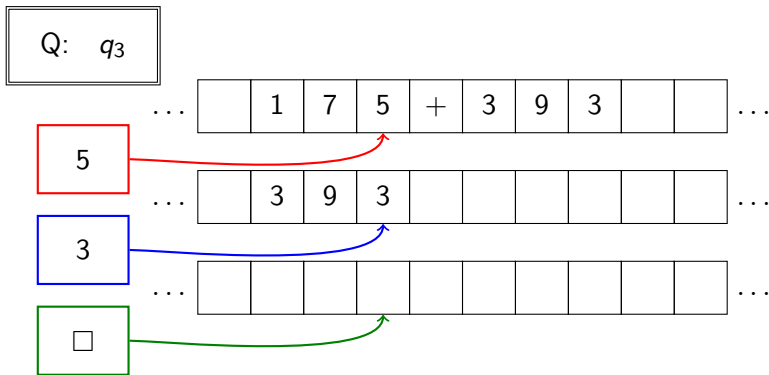
Likums:  $(q_2, +, \square) \rightarrow (q_3, (+, \triangleleft), (\square, \triangleleft), (\square, \triangleleft))$

# Tjūringa mašīna



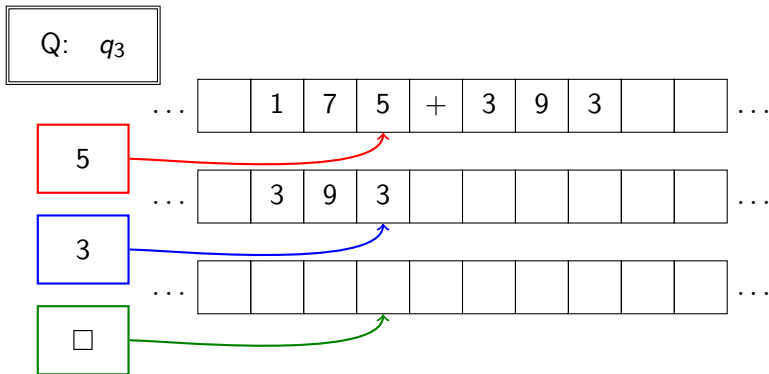
Likums:  $(q_2, +, \square) \rightarrow (q_3, (+, \triangleleft), (\square, \triangleleft), (\square, \triangleleft))$

# Tjūringa mašīna



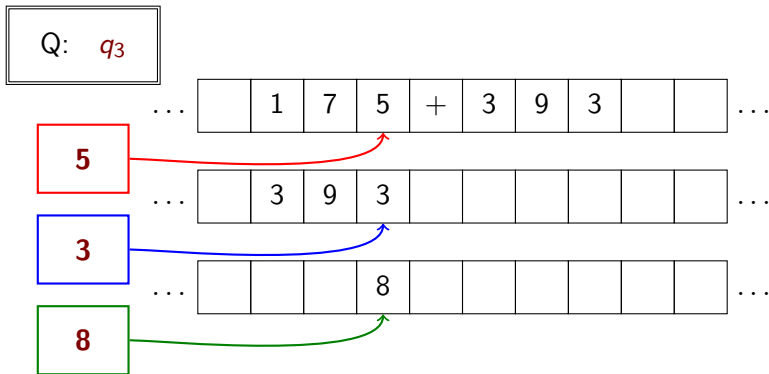
Likums:  $(q_2, +, \square) \rightarrow (q_3, (+, \triangleleft), (\square, \triangleleft), (\square, \triangleleft))$

# Tjūringa mašina



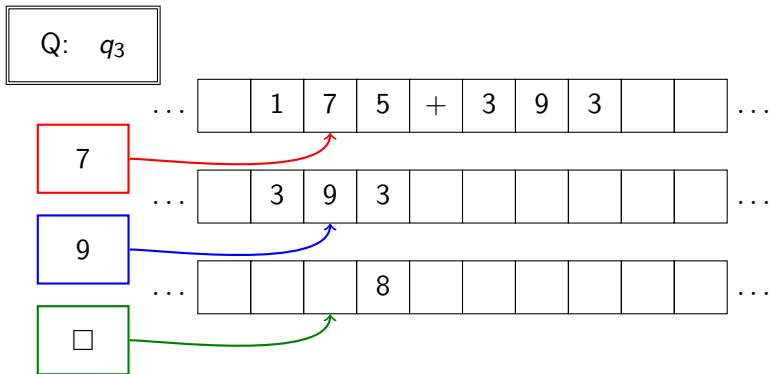
Likums:  $(q_3, n, m) \rightarrow (q_3, (n, \triangleleft), (m, \triangleleft), (n + m, \triangleleft))$   
vai  $(q_3, n, m) \rightarrow (q_4, (n, \triangleleft), (m, \triangleleft), (n + m - 10, \triangleleft))$

# Tjūringa mašina



Likums:  $(q_3, n, m) \rightarrow (q_3, (n, \triangleleft), (m, \triangleleft), (n + m, \triangleleft))$   
vai  $(q_3, n, m) \rightarrow (q_4, (n, \triangleleft), (m, \triangleleft), (n + m - 10, \triangleleft))$

# Tjūringa mašina



Likums:  $(q_3, n, m) \rightarrow (q_3, (n, \triangleleft), (m, \triangleleft), (n + m, \triangleleft))$

vai  $(q_3, n, m) \rightarrow (q_4, (n, \triangleleft), (m, \triangleleft), (n + m - 10, \triangleleft))$

## Tjūringa mašīna

$(q_3, n, m) \rightarrow (q_3, (n, \triangleleft), (m, \triangleleft), (n + m, \triangleleft))$  un

$(q_3, n, m) \rightarrow (q_4, (n, \triangleleft), (m, \triangleleft), (n + m - 10, \triangleleft))$

patiesībā **100** likumi:

$$1 \quad (q_3, 0, 0) \rightarrow (q_3, (0, \triangleleft), (0, \triangleleft), (0, \triangleleft))$$

$$2 \quad (q_3, 0, 1) \rightarrow (q_3, (0, \triangleleft), (1, \triangleleft), (1, \triangleleft))$$

$$3 \quad (q_3, 0, 2) \rightarrow (q_3, (0, \triangleleft), (2, \triangleleft), (2, \triangleleft))$$

...

$$45 \quad (q_3, 4, 4) \rightarrow (q_3, (4, \triangleleft), (4, \triangleleft), (8, \triangleleft))$$

$$46 \quad (q_3, 4, 5) \rightarrow (q_3, (4, \triangleleft), (5, \triangleleft), (9, \triangleleft))$$

$$47 \quad (q_3, 4, 6) \rightarrow (q_4, (4, \triangleleft), (6, \triangleleft), (0, \triangleleft))$$

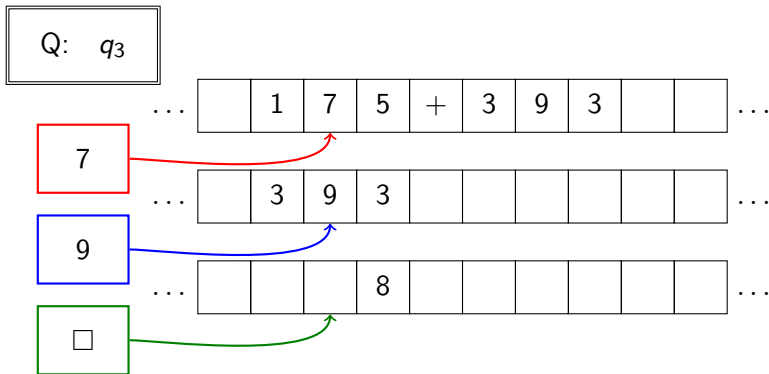
$$48 \quad (q_3, 4, 7) \rightarrow (q_4, (4, \triangleleft), (7, \triangleleft), (1, \triangleleft))$$

...

$$99 \quad (q_3, 9, 8) \rightarrow (q_4, (9, \triangleleft), (8, \triangleleft), (7, \triangleleft))$$

$$100 \quad (q_3, 9, 9) \rightarrow (q_4, (9, \triangleleft), (9, \triangleleft), (8, \triangleleft))$$

# Tjūringa mašina

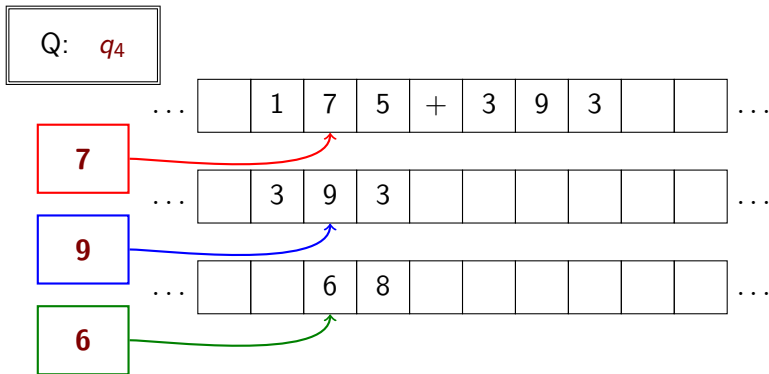


Likums:  $(q_3, n, m) \rightarrow (q_3, (n, \triangleleft), (m, \triangleleft), (n + m, \triangleleft))$

vai  $(q_3, n, m) \rightarrow (q_4, (n, \triangleleft), (m, \triangleleft), (n + m - 10, \triangleleft))$

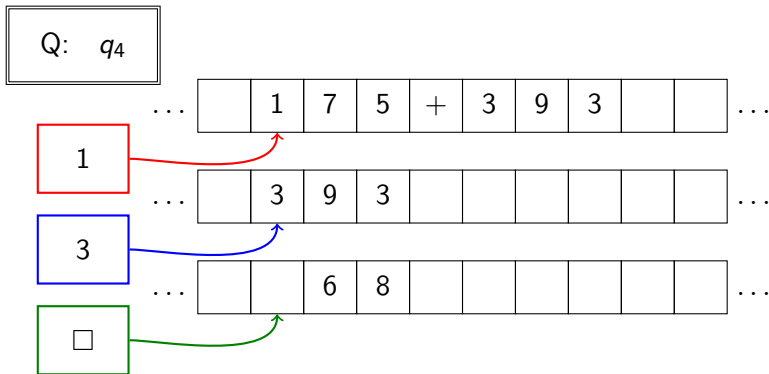


# Tjūringa mašina



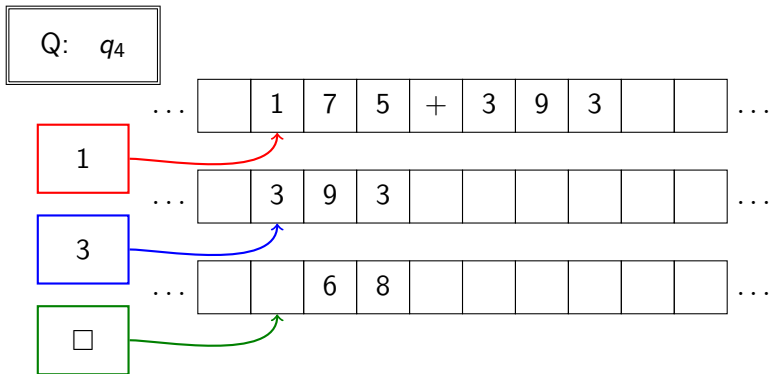
Likums:  $(q_3, n, m) \rightarrow (q_3, (n, \triangleleft), (m, \triangleleft), (n + m, \triangleleft))$   
vai  $(q_3, n, m) \rightarrow (q_4, (n, \triangleleft), (m, \triangleleft), (n + m - 10, \triangleleft))$

# Tjūringa mašina



Likums:  $(q_3, n, m) \rightarrow (q_3, (n, \triangleleft), (m, \triangleleft), (n + m, \triangleleft))$   
vai  $(q_3, n, m) \rightarrow (q_4, (n, \triangleleft), (m, \triangleleft), (n + m - 10, \triangleleft))$

# Tjūringa mašina



Likums:  $(q_4, n, m) \rightarrow (q_3, (n, \triangleleft), (m, \triangleleft), (n + m + 1, \triangleleft))$   
vai  $(q_4, n, m) \rightarrow (q_4, (n, \triangleleft), (m, \triangleleft), (n + m - 9, \triangleleft))$

## Tjūringa mašīna

$(q_4, n, m) \longrightarrow (q_3, (n, \triangleleft), (m, \triangleleft), (n + m + 1, \triangleleft))$  un

$(q_4, n, m) \longrightarrow (q_4, (n, \triangleleft), (m, \triangleleft), (n + m - 9, \triangleleft))$

patiesībā atkal **100** likumi:

$$1 \quad (q_4, 0, 0) \longrightarrow (q_3, (0, \triangleleft), (0, \triangleleft), (1, \triangleleft))$$

$$2 \quad (q_4, 0, 1) \longrightarrow (q_3, (0, \triangleleft), (1, \triangleleft), (2, \triangleleft))$$

$$3 \quad (q_4, 0, 2) \longrightarrow (q_3, (0, \triangleleft), (2, \triangleleft), (3, \triangleleft))$$

...

$$45 \quad (q_4, 4, 4) \longrightarrow (q_3, (4, \triangleleft), (4, \triangleleft), (9, \triangleleft))$$

$$46 \quad (q_4, 4, 5) \longrightarrow (q_4, (4, \triangleleft), (5, \triangleleft), (0, \triangleleft))$$

$$47 \quad (q_4, 4, 6) \longrightarrow (q_4, (4, \triangleleft), (6, \triangleleft), (1, \triangleleft))$$

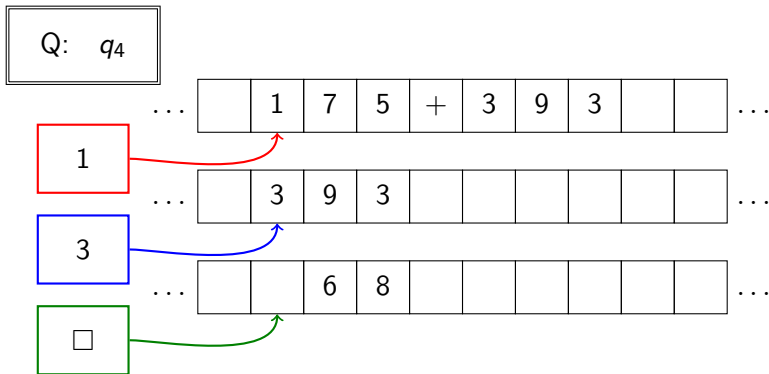
$$48 \quad (q_4, 4, 7) \longrightarrow (q_4, (4, \triangleleft), (7, \triangleleft), (2, \triangleleft))$$

...

$$99 \quad (q_4, 9, 8) \longrightarrow (q_4, (9, \triangleleft), (8, \triangleleft), (8, \triangleleft))$$

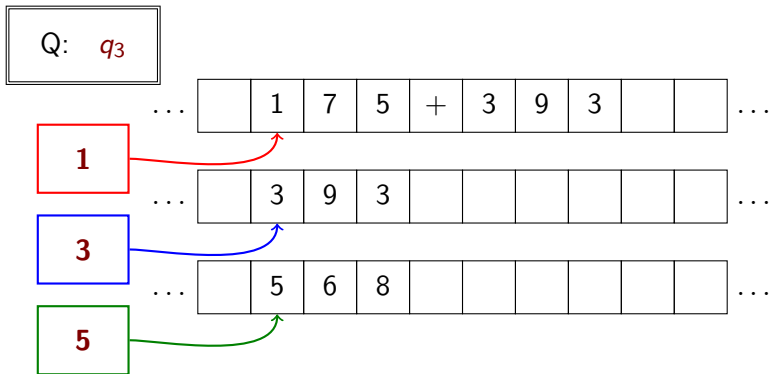
$$100 \quad (q_4, 9, 9) \longrightarrow (q_4, (9, \triangleleft), (9, \triangleleft), (9, \triangleleft))$$

# Tjūringa mašina



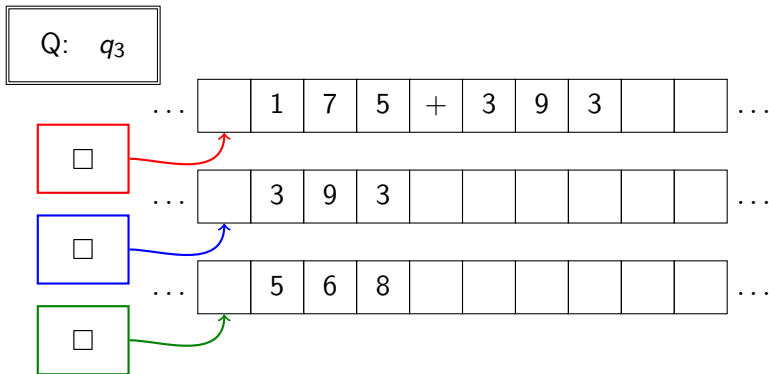
Likums:  $(q_4, n, m) \rightarrow (q_3, (n, \triangleleft), (m, \triangleleft), (n + m + 1, \triangleleft))$   
vai  $(q_4, n, m) \rightarrow (q_4, (n, \triangleleft), (m, \triangleleft), (n + m - 9, \triangleleft))$

# Tjūringa mašina



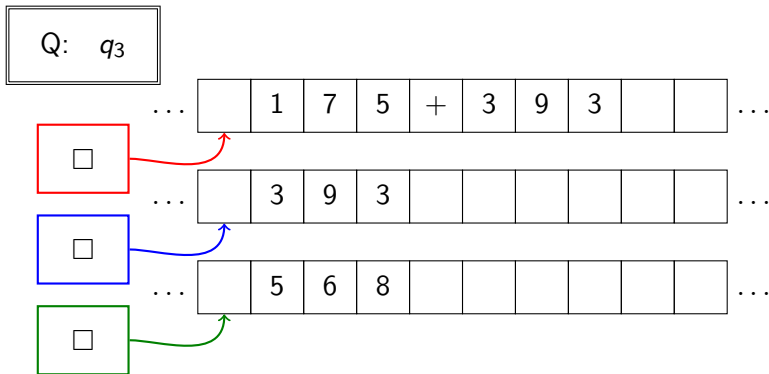
Likums:  $(q_4, n, m) \rightarrow (q_3, (n, \triangleleft), (m, \triangleleft), (n + m + 1, \triangleleft))$   
vai  $(q_4, n, m) \rightarrow (q_4, (n, \triangleleft), (m, \triangleleft), (n + m - 9, \triangleleft))$

# Tjūringa mašina



Likums:  $(q_4, n, m) \rightarrow (q_3, (n, \triangleleft), (m, \triangleleft), (n + m + 1, \triangleleft))$   
 vai  $(q_4, n, m) \rightarrow (q_4, (n, \triangleleft), (m, \triangleleft), (n + m - 9, \triangleleft))$

# Tjūringa mašina

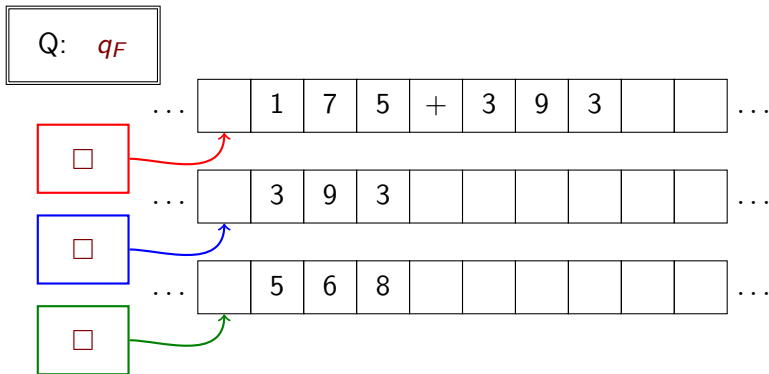


Likums:  $(q_3, \square, \square) \rightarrow (q_F, (\square, \Delta), (\square, \Delta), (\square, \Delta))$

vai  $(q_4, \square, \square) \rightarrow (q_F, (\square, \Delta), (\square, \Delta), (1, \Delta))$



# Tjūringa mašina



Likums:  $(q_3, \square, \square) \rightarrow (q_F, (\square, \Delta), (\square, \Delta), (\square, \Delta))$

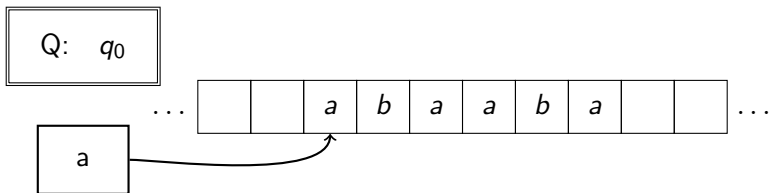
vai  $(q_4, \square, \square) \rightarrow (q_F, (\square, \Delta), (\square, \Delta), (1, \Delta))$

# Tjūringa mašīna

Kopā 245 likumi, kompakti uzrakstāmi kā

- 1  $(q_0, n, \square) \longrightarrow (q_0, (n, \triangleright), (\square, \Delta), (\square, \Delta))$
- 2  $(q_0, +, \square) \longrightarrow (q_1, (+, \triangleright), (\square, \Delta), (\square, \Delta))$
- 3  $(q_1, n, \square) \longrightarrow (q_1, (n, \triangleright), (n, \triangleright), (\square, \triangleright))$
- 4  $(q_1, \square, \square) \longrightarrow (q_2, (\square, \triangleleft), (\square, \Delta), (\square, \Delta))$
- 5  $(q_2, n, \square) \longrightarrow (q_2, (n, \triangleleft), (\square, \Delta), (\square, \Delta))$
- 6  $(q_2, +, \square) \longrightarrow (q_3, (+, \triangleleft), (\square, \triangleleft), (\square, \triangleleft))$
- 7  $(q_3, n, m) \longrightarrow (q_3, (n, \triangleleft), (m, \triangleleft), (n + m, \triangleleft))$
- 8  $(q_3, n, m) \longrightarrow (q_4, (n, \triangleleft), (m, \triangleleft), (n + m - 10, \triangleleft))$
- 9  $(q_4, n, m) \longrightarrow (q_3, (n, \triangleleft), (m, \triangleleft), (n + m + 1, \triangleleft))$
- 10  $(q_4, n, m) \longrightarrow (q_4, (n, \triangleleft), (m, \triangleleft), (n + m - 9, \triangleleft))$
- 11  $(q_3, \square, \square) \longrightarrow (q_F, (\square, \Delta), (\square, \Delta), (\square, \Delta))$
- 12  $(q_4, \square, \square) \longrightarrow (q_F, (\square, \Delta), (\square, \Delta), (1, \Delta))$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

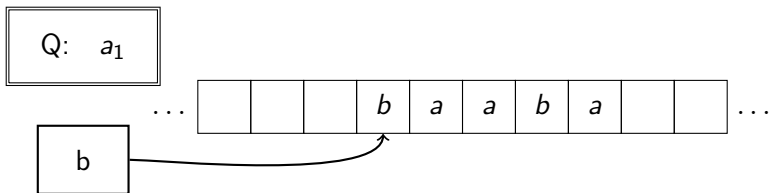
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

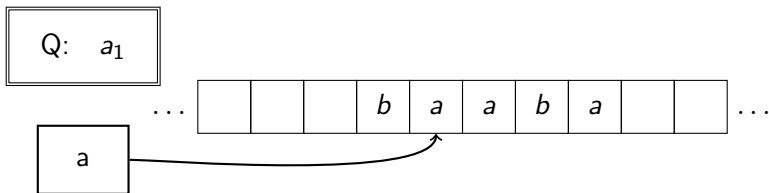
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

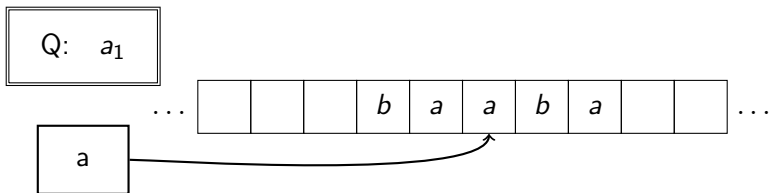
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

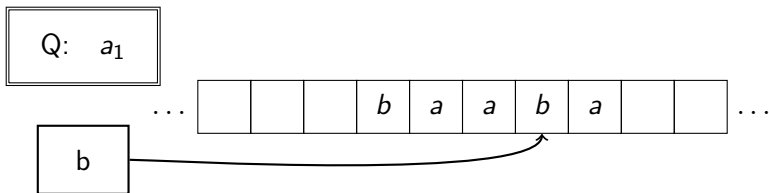
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

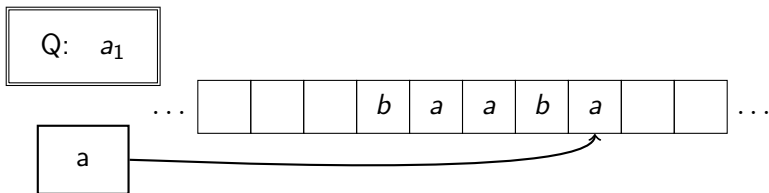
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

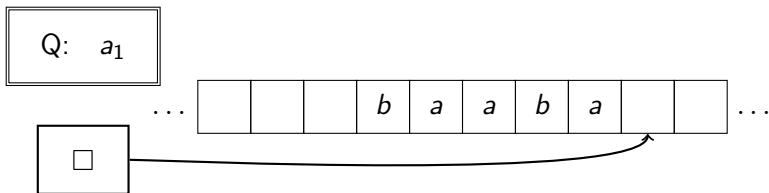
$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$



# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

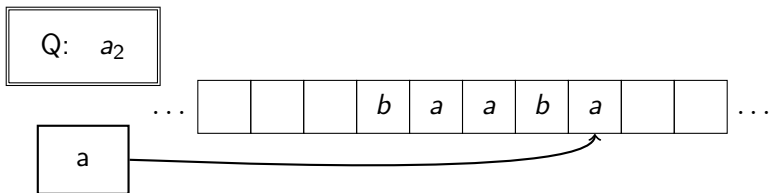
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

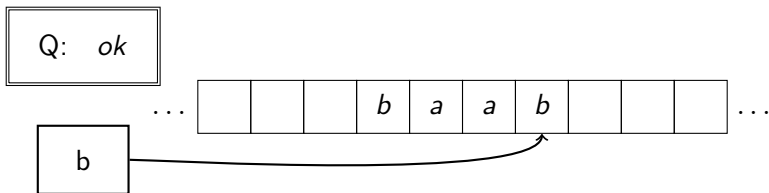
# Tjūringa mašina



$(q_0, \square) \rightarrow (q_F, (1, \Delta))$   
 $(q_0, a) \rightarrow (a_1, (\square, \triangleright))$   
 $(q_0, b) \rightarrow (b_1, (\square, \triangleright))$   
 $(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$   
 $(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$   
 $(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$   
 $(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$   
 $(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$   
 $(ok, \square) \rightarrow (q_0, (\square, \triangleright))$

**$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$**   
 $(a_2, b) \rightarrow (ne, (\square, \triangleleft))$   
 $(a_2, \square) \rightarrow (q_F, (1, \Delta))$   
 $(b_2, a) \rightarrow (ne, (\square, \triangleleft))$   
 $(b_2, b) \rightarrow (ok, (\square, \triangleleft))$   
 $(b_2, \square) \rightarrow (q_F, (1, \Delta))$   
 $(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$   
 $(ne, \square) \rightarrow (q_F, (0, \Delta))$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

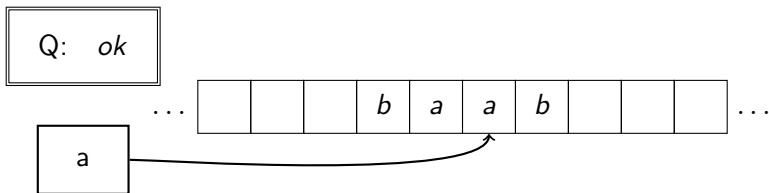
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

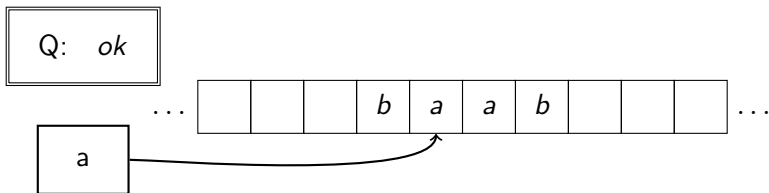
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

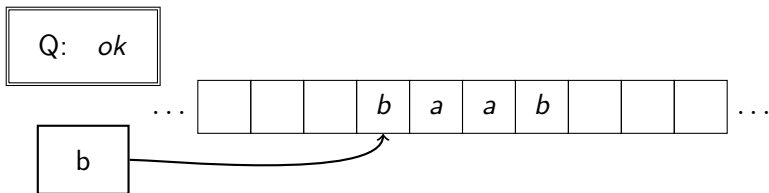
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

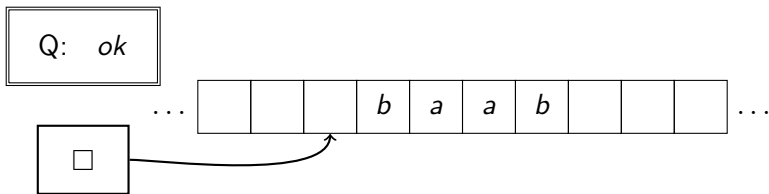
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

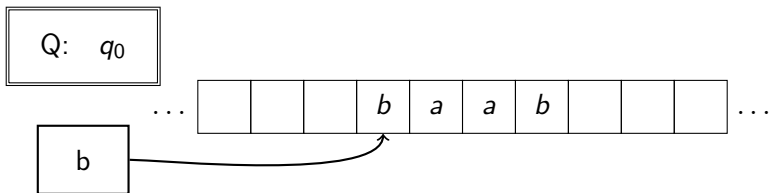
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

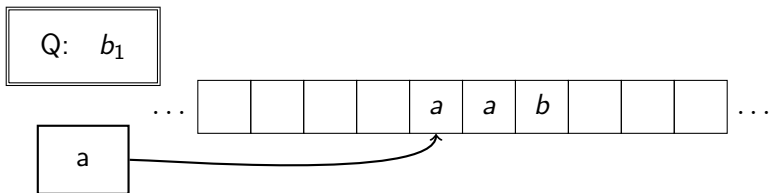
$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$



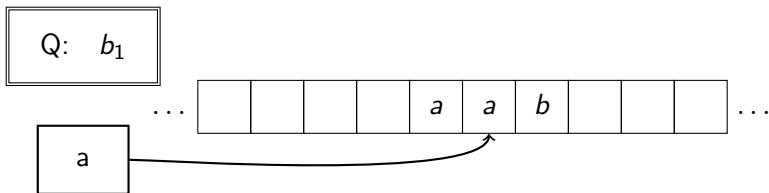
# Tjūringa mašina



$(q_0, \square) \rightarrow (q_F, (1, \Delta))$   
 $(q_0, a) \rightarrow (a_1, (\square, \triangleright))$   
 $(q_0, b) \rightarrow (b_1, (\square, \triangleright))$   
 $(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$   
 $(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$   
 **$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$**   
 $(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$   
 $(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$   
 $(ok, \square) \rightarrow (q_0, (\square, \triangleright))$

$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$   
 $(a_2, b) \rightarrow (ne, (\square, \triangleleft))$   
 $(a_2, \square) \rightarrow (q_F, (1, \Delta))$   
 $(b_2, a) \rightarrow (ne, (\square, \triangleleft))$   
 $(b_2, b) \rightarrow (ok, (\square, \triangleleft))$   
 $(b_2, \square) \rightarrow (q_F, (1, \Delta))$   
 $(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$   
 $(ne, \square) \rightarrow (q_F, (0, \Delta))$

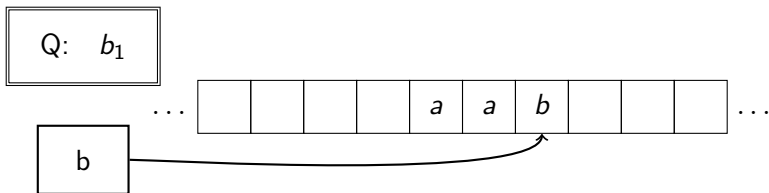
## Tjūringa mašina



$(q_0, \square) \rightarrow (q_F, (1, \Delta))$   
 $(q_0, a) \rightarrow (a_1, (\square, \triangleright))$   
 $(q_0, b) \rightarrow (b_1, (\square, \triangleright))$   
 $(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$   
 $(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$   
 **$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$**   
 $(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$   
 $(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$   
 $(ok, \square) \rightarrow (q_0, (\square, \triangleright))$

$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$   
 $(a_2, b) \rightarrow (ne, (\square, \triangleleft))$   
 $(a_2, \square) \rightarrow (q_F, (1, \Delta))$   
 $(b_2, a) \rightarrow (ne, (\square, \triangleleft))$   
 $(b_2, b) \rightarrow (ok, (\square, \triangleleft))$   
 $(b_2, \square) \rightarrow (q_F, (1, \Delta))$   
 $(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$   
 $(ne, \square) \rightarrow (q_F, (0, \Delta))$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

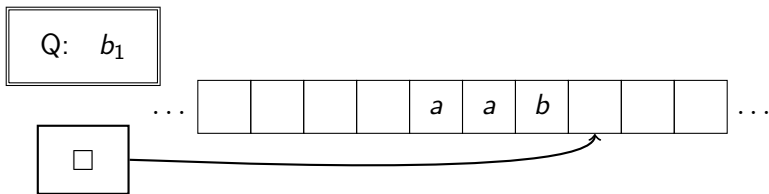
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

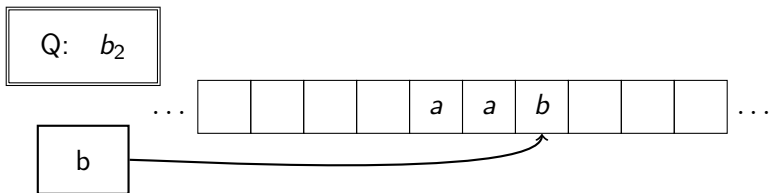
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

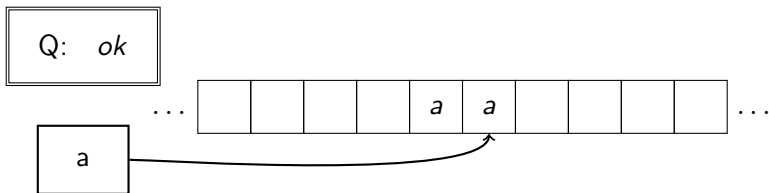
# Tjūringa mašina



$(q_0, \square) \rightarrow (q_F, (1, \Delta))$   
 $(q_0, a) \rightarrow (a_1, (\square, \triangleright))$   
 $(q_0, b) \rightarrow (b_1, (\square, \triangleright))$   
 $(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$   
 $(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$   
 $(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$   
 $(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$   
 $(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$   
 $(ok, \square) \rightarrow (q_0, (\square, \triangleright))$

$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$   
 $(a_2, b) \rightarrow (ne, (\square, \triangleleft))$   
 $(a_2, \square) \rightarrow (q_F, (1, \Delta))$   
 $(b_2, a) \rightarrow (ne, (\square, \triangleleft))$   
 **$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$**   
 $(b_2, \square) \rightarrow (q_F, (1, \Delta))$   
 $(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$   
 $(ne, \square) \rightarrow (q_F, (0, \Delta))$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

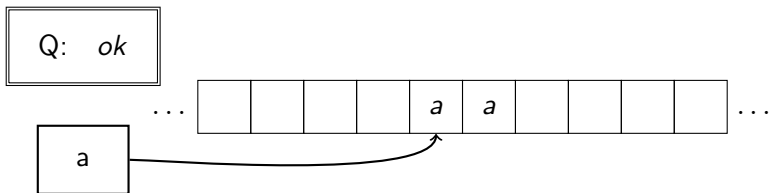
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

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# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

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$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

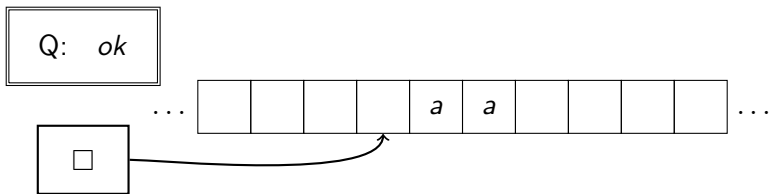
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

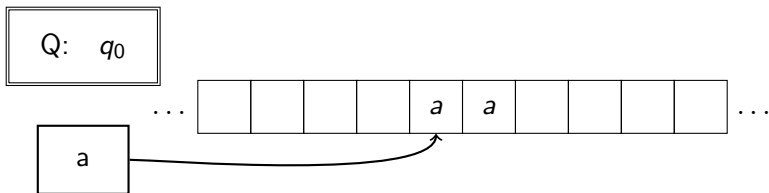
$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$



# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

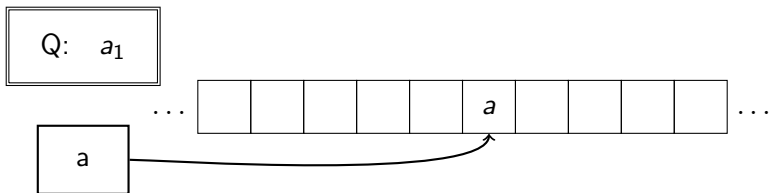
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

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# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

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$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

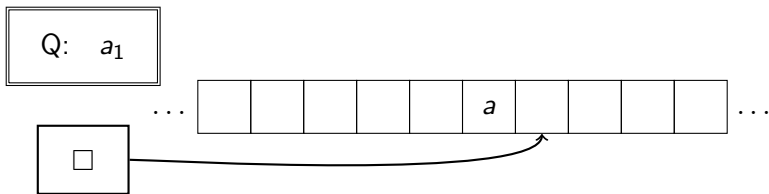
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

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$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

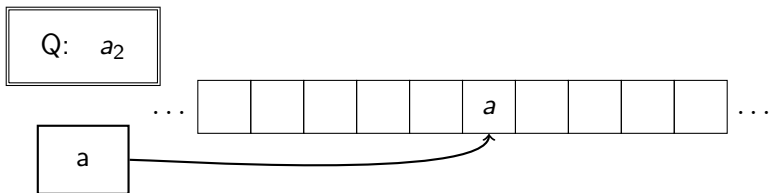
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

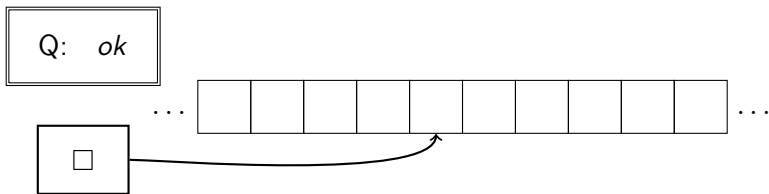
# Tjūringa mašina



$(q_0, \square) \rightarrow (q_F, (1, \Delta))$   
 $(q_0, a) \rightarrow (a_1, (\square, \triangleright))$   
 $(q_0, b) \rightarrow (b_1, (\square, \triangleright))$   
 $(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$   
 $(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$   
 $(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$   
 $(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$   
 $(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$   
 $(ok, \square) \rightarrow (q_0, (\square, \triangleright))$

**$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$**   
 $(a_2, b) \rightarrow (ne, (\square, \triangleleft))$   
 $(a_2, \square) \rightarrow (q_F, (1, \Delta))$   
 $(b_2, a) \rightarrow (ne, (\square, \triangleleft))$   
 $(b_2, b) \rightarrow (ok, (\square, \triangleleft))$   
 $(b_2, \square) \rightarrow (q_F, (1, \Delta))$   
 $(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$   
 $(ne, \square) \rightarrow (q_F, (0, \Delta))$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

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$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

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$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

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$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

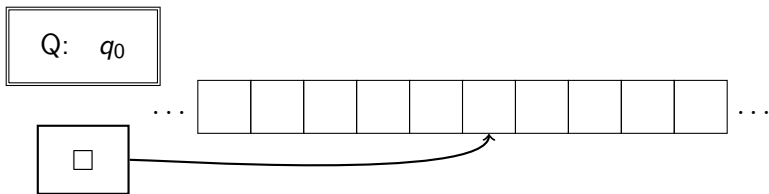
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

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# Tjūringa mašina



**$(q_0, \square) \rightarrow (q_F, (1, \Delta))$**

$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$

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$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$

$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$

$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$

$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$

$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$

$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$

$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$

$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$

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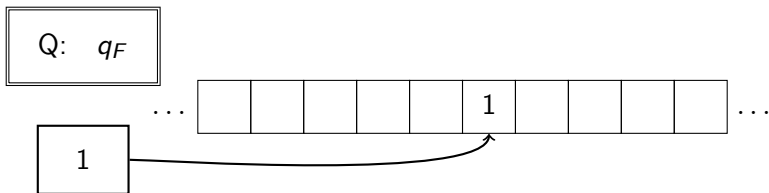
$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$

$(b_2, \square) \rightarrow (q_F, (1, \Delta))$

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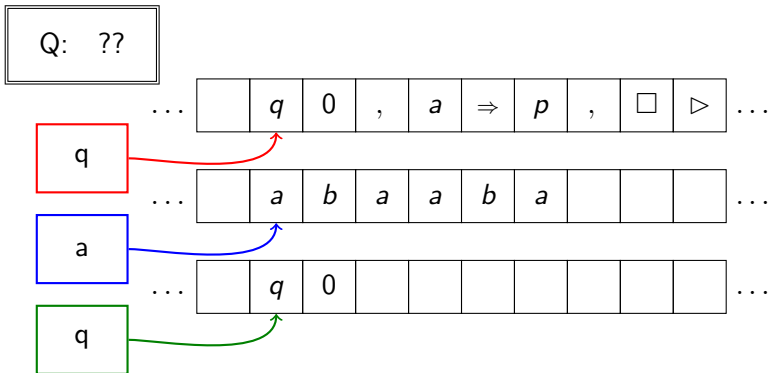
# Tjūringa mašina



$(q_0, \square) \rightarrow (q_F, (1, \Delta))$   
 $(q_0, a) \rightarrow (a_1, (\square, \triangleright))$   
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 $(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$   
 $(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$   
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 $(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$   
 $(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$   
 $(ok, \square) \rightarrow (q_0, (\square, \triangleright))$

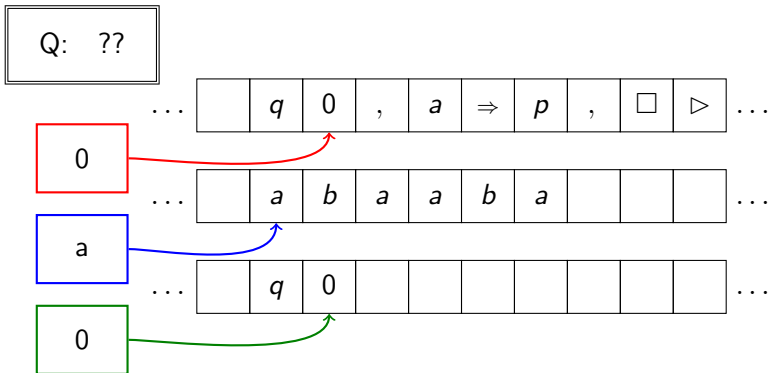
$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$   
 $(a_2, b) \rightarrow (ne, (\square, \triangleleft))$   
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# Universālā Tjūringa mašīna

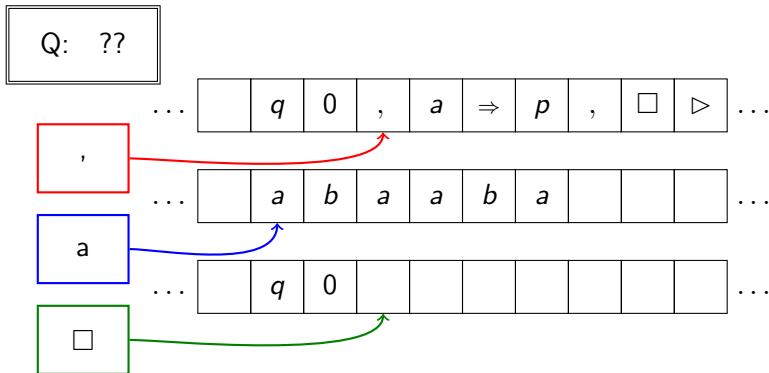




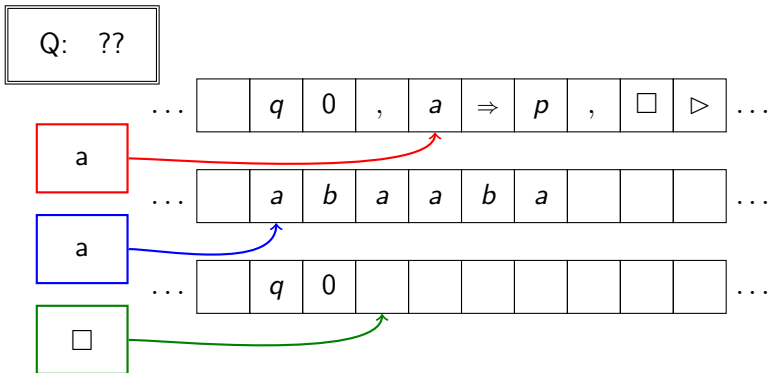
# Universālā Tjūringa mašīna



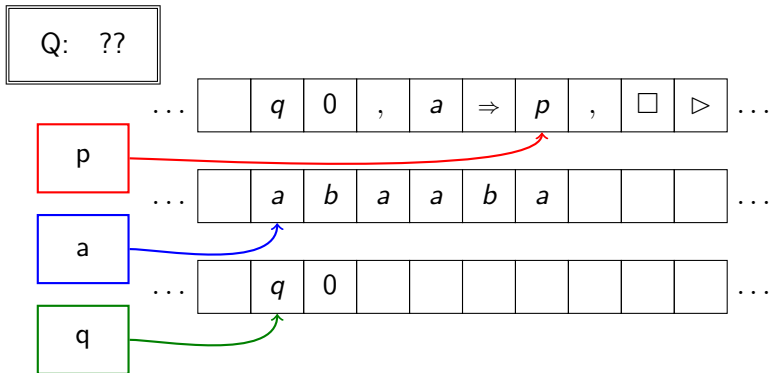
# Universālā Tjūringa mašīna



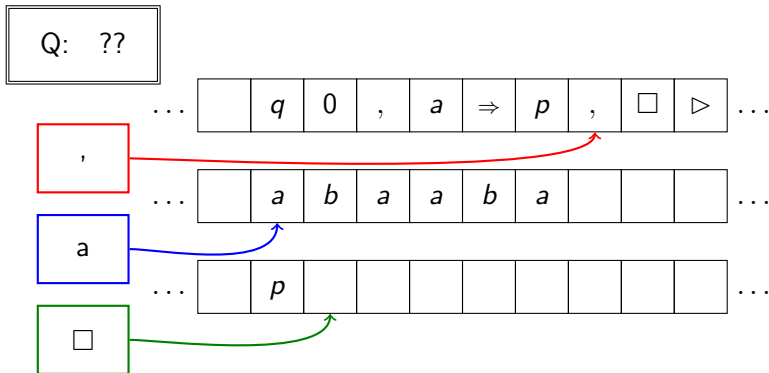
# Universālā Tjūringa mašīna



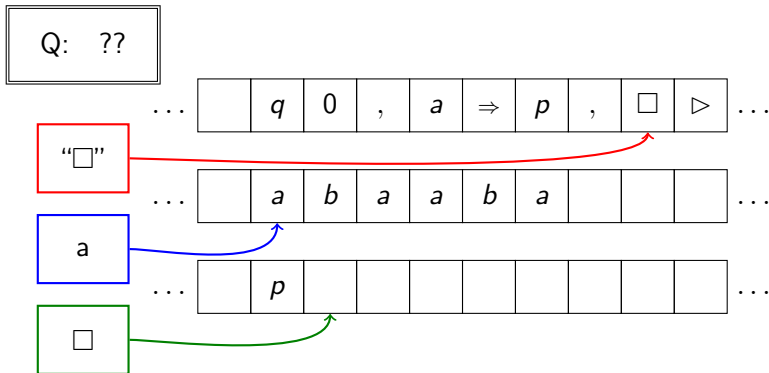
# Universālā Tjūringa mašīna



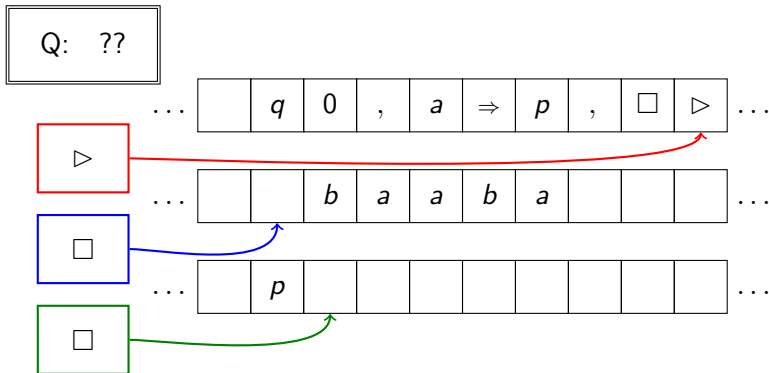
# Universālā Tjūringa mašīna



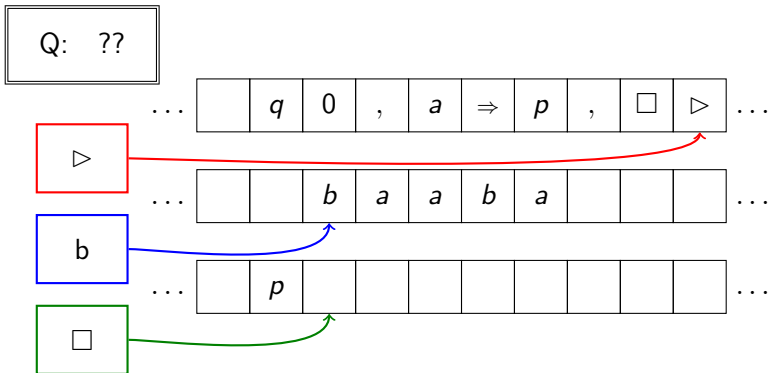
# Universālā Tjūringa mašīna



# Universālā Tjūringa mašīna

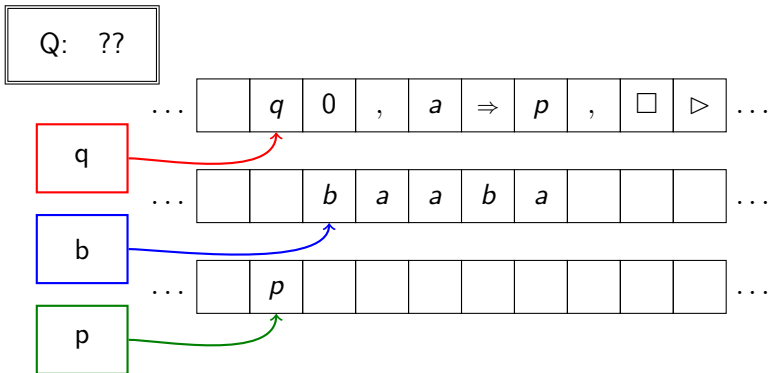


# Universālā Tjūringa mašīna





# Universālā Tjūringa mašīna



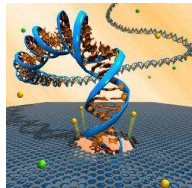
# Čerča-Tjūringa tēze

- Modernie datori atraduši ļoti plašu pielietojumu



Modernais dators

- Bioloģiskie datori izmanto bioloģiski iegūtas molekulu sistēmas, piemēram - DNS un proteīnus, lai veiktu skaitļošanas aprēķinus.



Bioloģiskais (DNS) dators

# Apstāšanās problēma

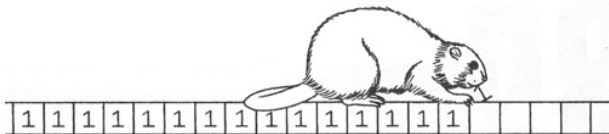
- Dažas Tjūringa mašīnas nekad neapstājas ...

# Apstāšanās problēma

- Dažas Tjūringa mašīnas nekad neapstājas . . .
  - un vispārīgā gadījumā nevar pateikt, vai dota mašīna apstāsies, vai nē.

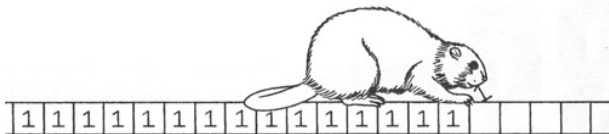
# Apstāšanās problēma

- Dažas Tjūringa mašīnas nekad neapstājas . . .
  - un vispārīgā gadījumā nevar pateikt, vai dota mašīna apstāsies, vai nē.
- **Uzcītīgais bebrs** ir Tjūringa mašīna, kas cenšas aizpildīt pēc iespējas lielu lentas apgabalu, bet tomēr apstājas.



# Apstāšanās problēma

- Dažas Tjūringa mašīnas nekad neapstājas . . .
  - un vispārīgā gadījumā nevar pateikt, vai dota mašīna apstāsies, vai nē.
- **Uzcītīgais bebrs** ir Tjūringa mašīna, kas cenšas aizpildīt pēc iespējas lielu lentas apgabalu, bet tomēr apstājas.
  - Vispārīgā gadījumā nevar arī pateikt, kurš no bebriem ir visuzcītīgākais.



# Diofanta vienādojumi

- Diofanta vienādojums ir

$$P(x_1, x_2, \dots, x_n) = 0,$$

kur  $P(x_1, x_2, \dots, x_n)$  ir domāts polinoms ar mainīgajiem  $x_1, x_2, \dots, x_n$  un veseliem koeficientiem.

- Laikam slavenākais Diofanta vienādojuma piemērs:

$$x^2 + y^2 = z^2$$

Piemēram,  $3^2 + 4^2 = 5^2$  bet arī  $5^2 + 12^2 = 13^2$ .

# Diofanta vienādojumi

- Diofanta vienādojums ir

$$P(x_1, x_2, \dots, x_n) = 0,$$

kur  $P(x_1, x_2, \dots, x_n)$  ir domāts polinoms ar mainīgajiem  $x_1, x_2, \dots, x_n$  un veseliem koeficientiem.

- Laikam slavenākais Diofanta vienādojuma piemērs:

$$x^2 + y^2 = z^2$$

Piemēram,  $3^2 + 4^2 = 5^2$  bet arī  $5^2 + 12^2 = 13^2$ .

- **Hilberta 10. problēma**, 1900.g.:  
atrast algoritmu, kas pēc dota Diofanta vienādojuma noskaidro, vai šim vienādojumam eksistē atrisinājums veselos skaitļos.



# Diofanta vienādojumi

- Diofanta vienādojums ir

$$P(x_1, x_2, \dots, x_n) = 0,$$

kur  $P(x_1, x_2, \dots, x_n)$  ir domāts polinoms ar mainīgajiem  $x_1, x_2, \dots, x_n$  un veseliem koeficientiem.

- Laikam slavenākais Diofanta vienādojuma piemērs:

$$x^2 + y^2 = z^2$$

Piemēram,  $3^2 + 4^2 = 5^2$  bet arī  $5^2 + 12^2 = 13^2$ .

- **Hilberta 10. problēma**, 1900.g.:  
atrast algoritmu, kas pēc dota Diofanta vienādojuma noskaidro, vai šim vienādojumam eksistē atrisinājums veselos skaitļos.
- **Teorēma** (J. Matijasevičs, 1970. g.) Neeksistē algoritms, kas pēc dota **patvaļīga** Diofanta vienādojuma noskaidro, vai šim vienādojumam eksistē atrisinājums veselos skaitļos.

# Kas ir sarežģīti?

Kā vieglāk sareizināt divus skaitļus?

$$137 \times 428 = ?$$

## Kas ir sarežģīti?

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$$137 \times 428 = ?$$

- Varam 137 reizes saskaitīt skaitli 428:

$$\begin{aligned} 137 * 428 &= 428 + 428 + 428 + 428 + 428 + 428 \\ &+ 428 + 428 + 428 + 428 + 428 + 428 + 428 + 428 + 428 + 428 \\ &+ 428 + 428 + 428 + 428 + 428 + 428 + 428 + 428 + 428 \\ &+ 428 + 428 + 428 + 428 + 428 + 428 + 428 + 428 + 428 \\ &+ 428 + 428 + 428 + 428 + 428 + 428 + 428 + \dots = ??? \end{aligned}$$

Viegls algoritms?

## Kas ir sarežģīti?

Kā vieglāk sareizināt divus skaitļus?

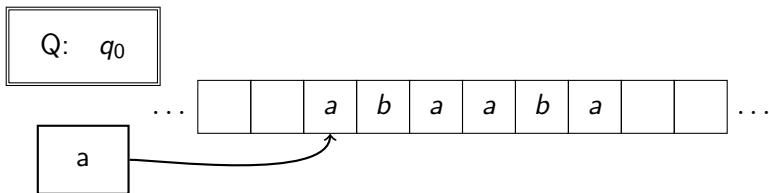
$$137 \times 428 = ?$$

- Varam reizināt stabiņā:

$$\begin{array}{r} \phantom{\times} \phantom{00} 137 \\ \times \phantom{00} 428 \\ \hline 1096 \\ + 274 \phantom{0} \\ + 548 \phantom{00} \\ \hline 58636 \end{array}$$

Esam atraduši atbildi:  $137 \times 428 = 58636$ .

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

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$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

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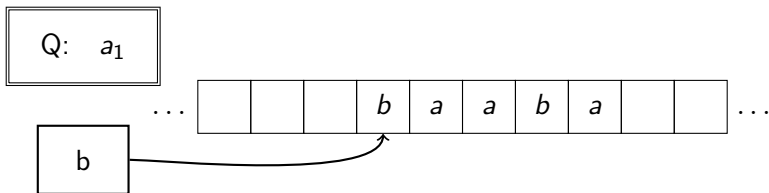
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

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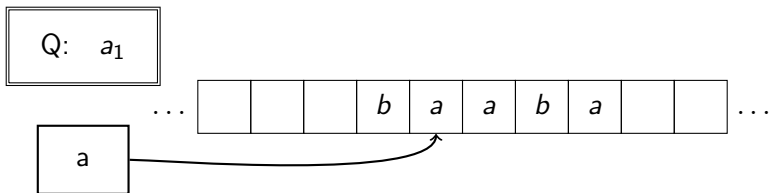
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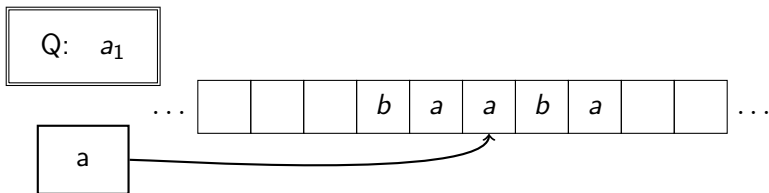
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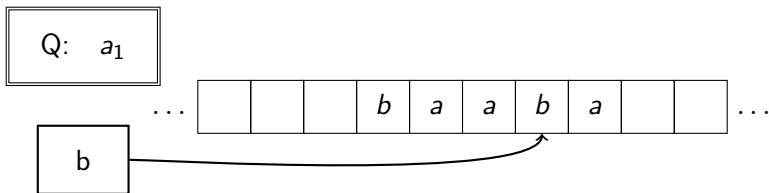
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## Tjūringa mašina



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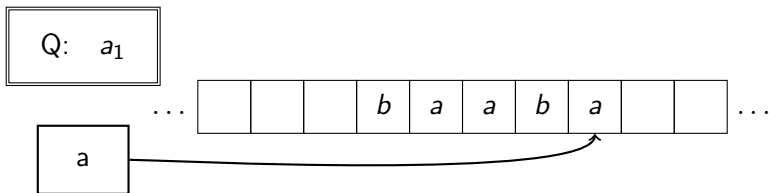
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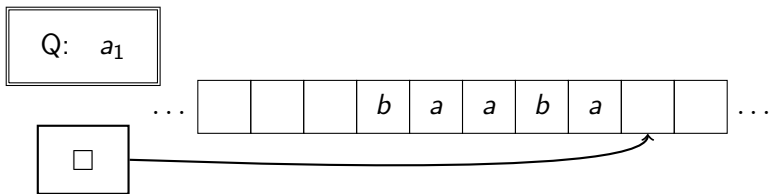
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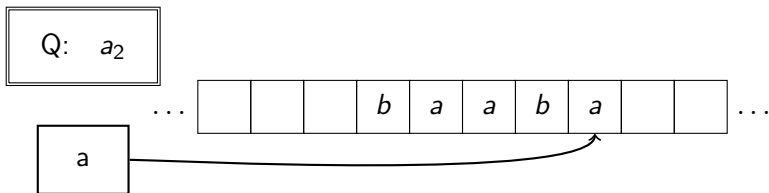
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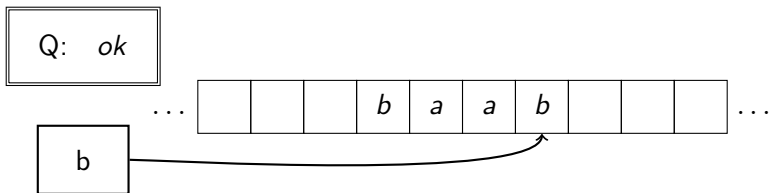
# Tjūringa mašina



$(q_0, \square) \rightarrow (q_F, (1, \Delta))$   
 $(q_0, a) \rightarrow (a_1, (\square, \triangleright))$   
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 $(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$   
 $(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$   
 $(ok, \square) \rightarrow (q_0, (\square, \triangleright))$

**$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$**   
 $(a_2, b) \rightarrow (ne, (\square, \triangleleft))$   
 $(a_2, \square) \rightarrow (q_F, (1, \Delta))$   
 $(b_2, a) \rightarrow (ne, (\square, \triangleleft))$   
 $(b_2, b) \rightarrow (ok, (\square, \triangleleft))$   
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# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

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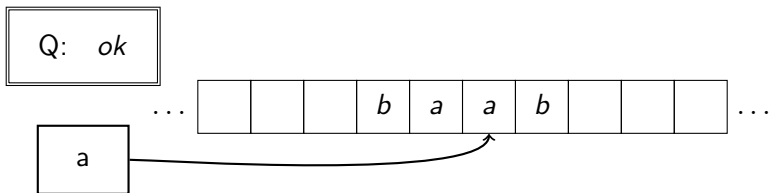
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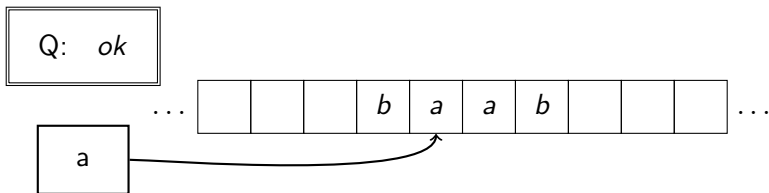
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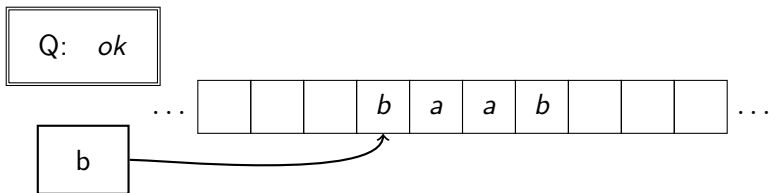
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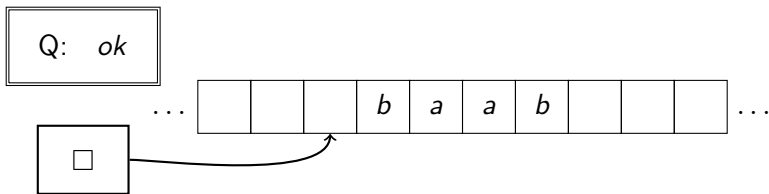
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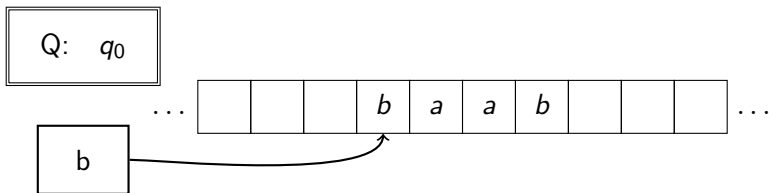
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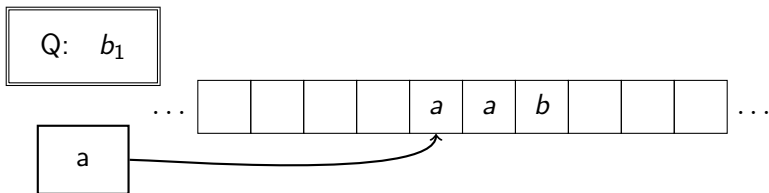
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$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

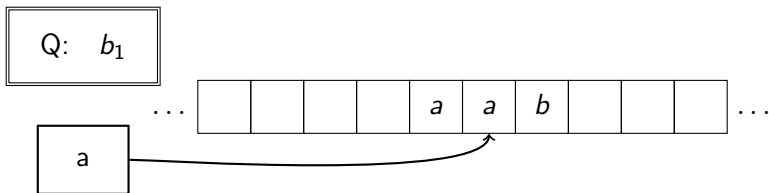
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

## Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

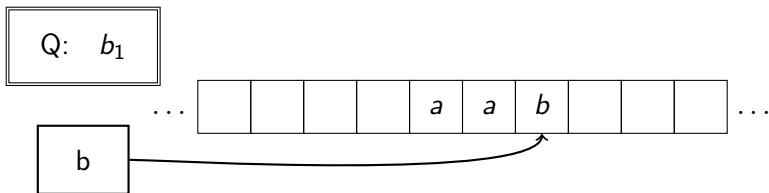
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

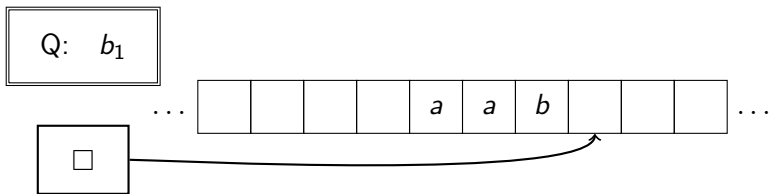
# Tjūringa mašina



$(q_0, \square) \rightarrow (q_F, (1, \Delta))$   
 $(q_0, a) \rightarrow (a_1, (\square, \triangleright))$   
 $(q_0, b) \rightarrow (b_1, (\square, \triangleright))$   
 $(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$   
 $(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$   
 **$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$**   
 $(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$   
 $(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$   
 $(ok, \square) \rightarrow (q_0, (\square, \triangleright))$

$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$   
 $(a_2, b) \rightarrow (ne, (\square, \triangleleft))$   
 $(a_2, \square) \rightarrow (q_F, (1, \Delta))$   
 $(b_2, a) \rightarrow (ne, (\square, \triangleleft))$   
 $(b_2, b) \rightarrow (ok, (\square, \triangleleft))$   
 $(b_2, \square) \rightarrow (q_F, (1, \Delta))$   
 $(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$   
 $(ne, \square) \rightarrow (q_F, (0, \Delta))$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \triangle))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \triangle))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

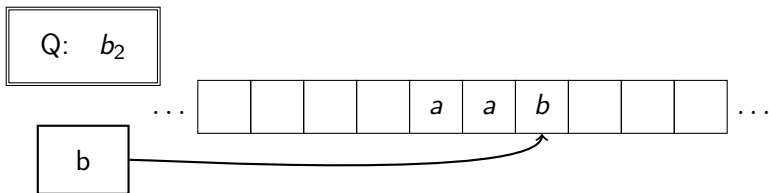
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \triangle))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \triangle))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

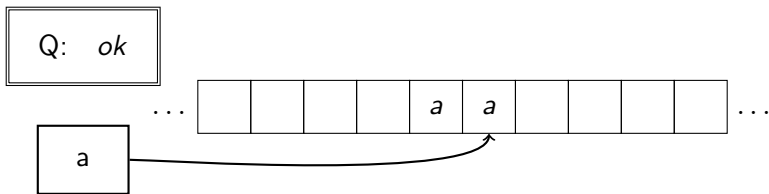
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \triangle))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \triangle))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

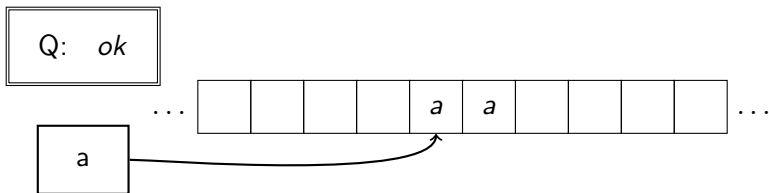
$$(b_2, \square) \rightarrow (q_F, (1, \triangle))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \triangle))$$



# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

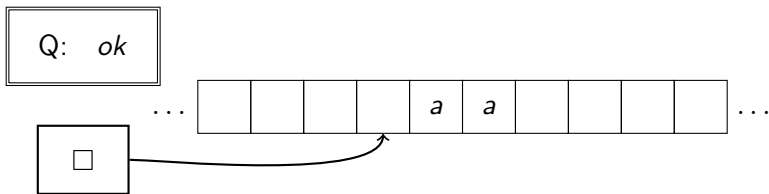
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

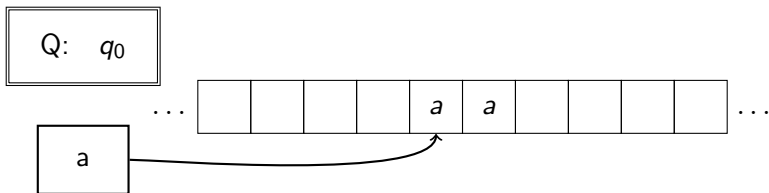
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

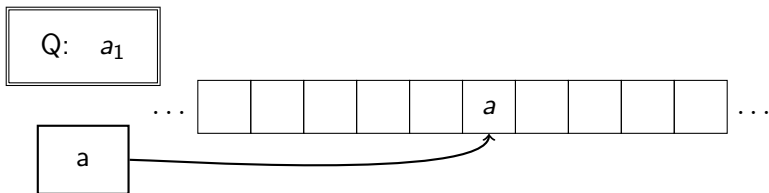
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

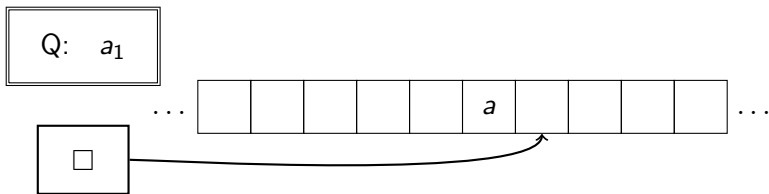
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

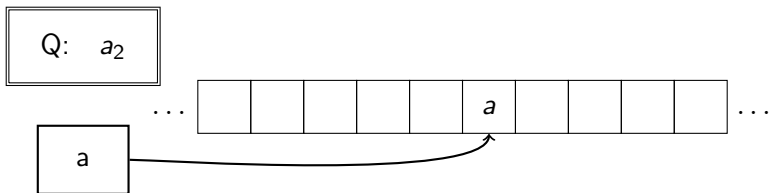
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$

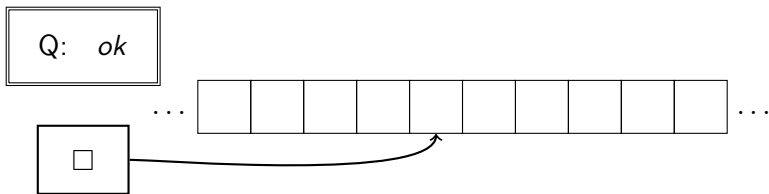
# Tjūringa mašina



$(q_0, \square) \rightarrow (q_F, (1, \Delta))$   
 $(q_0, a) \rightarrow (a_1, (\square, \triangleright))$   
 $(q_0, b) \rightarrow (b_1, (\square, \triangleright))$   
 $(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$   
 $(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$   
 $(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$   
 $(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$   
 $(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$   
 $(ok, \square) \rightarrow (q_0, (\square, \triangleright))$

$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$   
 $(a_2, b) \rightarrow (ne, (\square, \triangleleft))$   
 $(a_2, \square) \rightarrow (q_F, (1, \Delta))$   
 $(b_2, a) \rightarrow (ne, (\square, \triangleleft))$   
 $(b_2, b) \rightarrow (ok, (\square, \triangleleft))$   
 $(b_2, \square) \rightarrow (q_F, (1, \Delta))$   
 $(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$   
 $(ne, \square) \rightarrow (q_F, (0, \Delta))$

# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

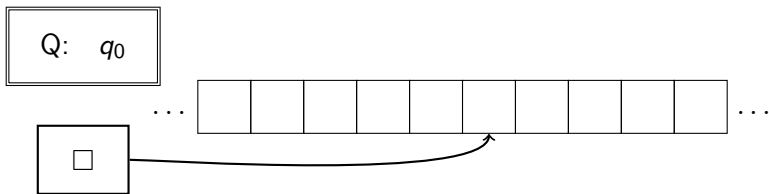
$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

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# Tjūringa mašina



$$(q_0, \square) \rightarrow (q_F, (1, \Delta))$$

$$(q_0, a) \rightarrow (a_1, (\square, \triangleright))$$

$$(q_0, b) \rightarrow (b_1, (\square, \triangleright))$$

$$(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$$

$$(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$$

$$(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$$

$$(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$$

$$(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$$

$$(ok, \square) \rightarrow (q_0, (\square, \triangleright))$$

$$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$$

$$(a_2, b) \rightarrow (ne, (\square, \triangleleft))$$

$$(a_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(b_2, a) \rightarrow (ne, (\square, \triangleleft))$$

$$(b_2, b) \rightarrow (ok, (\square, \triangleleft))$$

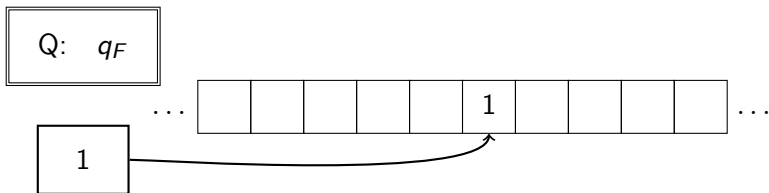
$$(b_2, \square) \rightarrow (q_F, (1, \Delta))$$

$$(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$$

$$(ne, \square) \rightarrow (q_F, (0, \Delta))$$



# Tjūringa mašina



$(q_0, \square) \rightarrow (q_F, (1, \Delta))$   
 $(q_0, a) \rightarrow (a_1, (\square, \triangleright))$   
 $(q_0, b) \rightarrow (b_1, (\square, \triangleright))$   
 $(a_1, a/b) \rightarrow (a_1, (a/b, \triangleright))$   
 $(a_1, \square) \rightarrow (a_2, (\square, \triangleleft))$   
 $(b_1, a/b) \rightarrow (b_1, (a/b, \triangleright))$   
 $(b_1, \square) \rightarrow (b_2, (\square, \triangleleft))$   
 $(ok, a/b) \rightarrow (ok, (a/b, \triangleleft))$   
 $(ok, \square) \rightarrow (q_0, (\square, \triangleright))$

$(a_2, a) \rightarrow (ok, (\square, \triangleleft))$   
 $(a_2, b) \rightarrow (ne, (\square, \triangleleft))$   
 $(a_2, \square) \rightarrow (q_F, (1, \Delta))$   
 $(b_2, a) \rightarrow (ne, (\square, \triangleleft))$   
 $(b_2, b) \rightarrow (ok, (\square, \triangleleft))$   
 $(b_2, \square) \rightarrow (q_F, (1, \Delta))$   
 $(ne, a/b) \rightarrow (ne, (\square, \triangleleft))$   
 $(ne, \square) \rightarrow (q_F, (0, \Delta))$

## Lielā O pieraksts

- Raksta  $f(x) = O(x^k)$ , ja eksistē pozitīva konstante  $C$  tāda, ka visiem  $x > x_0$  izpildās sakarība:

$$|f(x)| \leq C \cdot |x^k|.$$

- **Piemērs:**  $f(x) = 6x^4 - 2x^3 + 5$ .

Parādīsim, ka  $f(x) = O(x^4)$ . Izvēlamies par  $x_0 = 1$ . Tad

$$\begin{aligned} |6x^4 - 2x^3 + 5| &\leq 6x^4 + |2x^3| + 5 \\ &\leq 6x^4 + 2x^4 + 5x^4 \\ &\leq 13x^4. \end{aligned}$$

- Kad runā par algoritma sarežģītību pieņemts norādīt mazāko no klasēm, kam tas pieder.

## sarežģītības klase $\mathbf{P}$

- Sarežģītības klasei  $\mathbf{P}$  pieder visi algoritmi, kas pieder kādai no šīm kopām:

$$O(n), O(n^2), O(n^3), \dots, O(n^k), \dots$$

- Eksistē algoritmi, par kuriem varam pateikt, ka to sarežģītība ir starp kādām no šīm divām klasēm, piem.,

$$O(n) \subsetneq O(n \log n) \subsetneq O(n^2).$$

### Piemēri:

- 1 Divu skaitļu saskaitīšanas algoritms pieder klasei  $O(n)$ .
- 2 Simetrijas atpazīšanas algoritms (TM ar 1 lentu) pieder klasei  $O(n^2)$ .

# sarežģītības klase **NP** un NP-pilnas problēmas

- Ir daudz problēmu, kurām nav zināma “vienkārša” (polinomiāla) algoritma eksistence.
- Klase NP – problēmas, ko var “vienkārši” pārbaudīt.
- NP-pilnas problēmas ir sarežģītākās no NP problēmām.

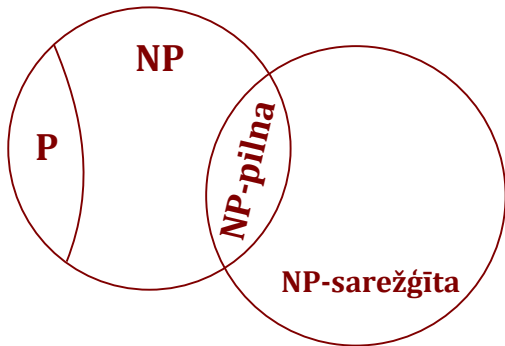


# NP-pilnas problēmas piemērs



att.: Mugursomas sapakošanas problēma

## Saikne starp P un NP klasēm



Saikne starp P un NP klasēm

$$P \stackrel{?}{=} NP$$

# 1. uzdevums

## 1. Uzdevums

Konstruēt Tjūringa mašīnu, kas saskaita divus vienāda garuma skaitļus, izmantojot tikai divas lentas. Par pamatu var izmantot programmu, kas saskaita izmantojot trīs lentas.



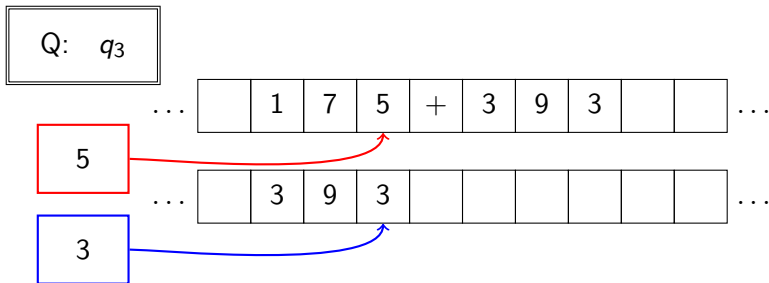
# 1. uzdevums

Likumi saskaitīšanai ar trīs lentām:

- 1  $(q_0, n, \square) \longrightarrow (q_0, (n, \triangleright), (\square, \Delta), (\square, \Delta))$
- 2  $(q_0, +, \square) \longrightarrow (q_1, (+, \triangleright), (\square, \Delta), (\square, \Delta))$
- 3  $(q_1, n, \square) \longrightarrow (q_1, (n, \triangleright), (n, \triangleright), (\square, \triangleright))$
- 4  $(q_1, \square, \square) \longrightarrow (q_2, (\square, \triangleleft), (\square, \Delta), (\square, \Delta))$
- 5  $(q_2, n, \square) \longrightarrow (q_2, (n, \triangleleft), (\square, \Delta), (\square, \Delta))$
- 6  $(q_2, +, \square) \longrightarrow (q_3, (+, \triangleleft), (\square, \triangleleft), (\square, \triangleleft))$
- 7  $(q_3, n, m) \longrightarrow (q_3, (n, \triangleleft), (m, \triangleleft), (n + m, \triangleleft))$
- 8  $(q_3, n, m) \longrightarrow (q_4, (n, \triangleleft), (m, \triangleleft), (n + m - 10, \triangleleft))$
- 9  $(q_4, n, m) \longrightarrow (q_3, (n, \triangleleft), (m, \triangleleft), (n + m + 1, \triangleleft))$
- 10  $(q_4, n, m) \longrightarrow (q_4, (n, \triangleleft), (m, \triangleleft), (n + m - 9, \triangleleft))$
- 11  $(q_3, \square, \square) \longrightarrow (q_F, (\square, \Delta), (\square, \Delta), (\square, \Delta))$
- 12  $(q_4, \square, \square) \longrightarrow (q_F, (\square, \Delta), (\square, \Delta), (1, \Delta))$

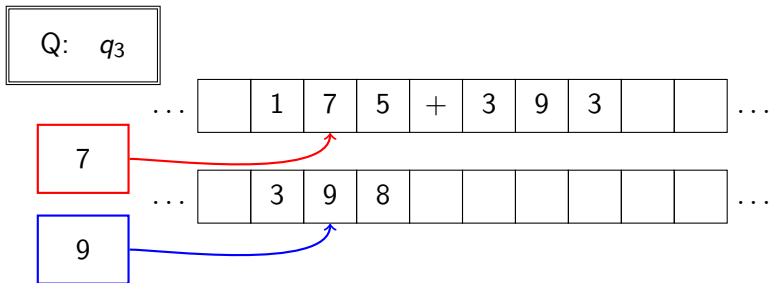
# 1. uzdevums — atrisinājums

Atrisinājumu raksta nevis uz trešās lentas, bet pāri otrai.



# 1. uzdevums — atrisinājums

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# 1. uzdevums — atrisinājums

Programma, kas saskaita izmantojot divas lentas izskatīsies sekojoši:

- ①  $(q_0, n, \square) \longrightarrow (q_0, (n, \triangleright), (\square, \Delta))$
- ②  $(q_0, +, \square) \longrightarrow (q_1, (+, \triangleright), (\square, \Delta))$
- ③  $(q_1, n, \square) \longrightarrow (q_1, (n, \triangleright), (n, \triangleright))$
- ④  $(q_1, \square, \square) \longrightarrow (q_2, (\square, \triangleleft), (\square, \Delta))$
- ⑤  $(q_2, n, \square) \longrightarrow (q_2, (n, \triangleleft), (\square, \Delta))$
- ⑥  $(q_2, +, \square) \longrightarrow (q_3, (+, \triangleleft), (\square, \triangleleft))$
- ⑦  $(q_3, n, m) \longrightarrow (q_3, (n, \triangleleft), (n + m, \triangleleft))$
- ⑧  $(q_3, n, m) \longrightarrow (q_4, (n, \triangleleft), (n + m - 10, \triangleleft))$
- ⑨  $(q_4, n, m) \longrightarrow (q_3, (n, \triangleleft), (n + m + 1, \triangleleft))$
- ⑩  $(q_4, n, m) \longrightarrow (q_4, (n, \triangleleft), (n + m - 9, \triangleleft))$
- ⑪  $(q_3, \square, \square) \longrightarrow (q_F, (\square, \Delta), (\square, \Delta))$
- ⑫  $(q_4, \square, \square) \longrightarrow (q_F, (\square, \Delta), (1, \Delta))$

## 2.uzdevums

2.uzdevums Dota Tjūringa mašīna ar 2 lentām un šādu likumu kopu:

$(q_0, n, \square)$	$\rightarrow$	$(q_0, (n, \triangleright), (\square, \triangle))$
$(q_0, \square, \square)$	$\rightarrow$	$(q_1, (\square, \triangleleft), (\square, \triangle))$
<hr/>		
$(q_1, 0, \square)$	$\rightarrow$	$(q_2, (0, \triangleleft), (\square, \triangle))$
$(q_1, 1, \square)$	$\rightarrow$	$(q_3, (1, \triangleleft), (\square, \triangle))$
$(q_1, 2, \square)$	$\rightarrow$	$(q_4, (2, \triangleleft), (\square, \triangle))$
$(q_1, 3, \square)$	$\rightarrow$	$(q_5, (3, \triangleleft), (\square, \triangle))$
$(q_1, 4, \square)$	$\rightarrow$	$(q_2, (4, \triangleleft), (\square, \triangle))$
$(q_1, 5, \square)$	$\rightarrow$	$(q_3, (5, \triangleleft), (\square, \triangle))$
$(q_1, 6, \square)$	$\rightarrow$	$(q_4, (6, \triangleleft), (\square, \triangle))$
$(q_1, 7, \square)$	$\rightarrow$	$(q_5, (7, \triangleleft), (\square, \triangle))$
$(q_1, 8, \square)$	$\rightarrow$	$(q_2, (8, \triangleleft), (\square, \triangle))$
$(q_1, 9, \square)$	$\rightarrow$	$(q_3, (9, \triangleleft), (\square, \triangle))$

## 2.uzdevums

$$\begin{aligned}(q_2, 0, \square) &\rightarrow (q_F, (0, \triangle), (0, \triangle)) \\(q_2, 1, \square) &\rightarrow (q_F, (1, \triangle), (2, \triangle)) \\(q_2, 2, \square) &\rightarrow (q_F, (2, \triangle), (0, \triangle)) \\(q_2, 3, \square) &\rightarrow (q_F, (3, \triangle), (2, \triangle)) \\(q_2, 4, \square) &\rightarrow (q_F, (4, \triangle), (0, \triangle)) \\(q_2, 5, \square) &\rightarrow (q_F, (5, \triangle), (2, \triangle)) \\(q_2, 6, \square) &\rightarrow (q_F, (6, \triangle), (0, \triangle)) \\(q_2, 7, \square) &\rightarrow (q_F, (7, \triangle), (2, \triangle)) \\(q_2, 8, \square) &\rightarrow (q_F, (8, \triangle), (0, \triangle)) \\(q_2, 9, \square) &\rightarrow (q_F, (9, \triangle), (2, \triangle)) \\(q_2, \square, \square) &\rightarrow (q_F, (\square, \triangle), (0, \triangle))\end{aligned}$$

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## 2.uzdevums

$$\begin{aligned}(q_3, 0, \square) &\rightarrow (q_F, (0, \triangle), (1, \triangle)) \\(q_3, 1, \square) &\rightarrow (q_F, (1, \triangle), (3, \triangle)) \\(q_3, 2, \square) &\rightarrow (q_F, (2, \triangle), (1, \triangle)) \\(q_3, 3, \square) &\rightarrow (q_F, (3, \triangle), (3, \triangle)) \\(q_3, 4, \square) &\rightarrow (q_F, (4, \triangle), (1, \triangle)) \\(q_3, 5, \square) &\rightarrow (q_F, (5, \triangle), (3, \triangle)) \\(q_3, 6, \square) &\rightarrow (q_F, (6, \triangle), (1, \triangle)) \\(q_3, 7, \square) &\rightarrow (q_F, (7, \triangle), (3, \triangle)) \\(q_3, 8, \square) &\rightarrow (q_F, (8, \triangle), (1, \triangle)) \\(q_3, 9, \square) &\rightarrow (q_F, (9, \triangle), (3, \triangle)) \\(q_3, \square, \square) &\rightarrow (q_F, (\square, \triangle), (1, \triangle))\end{aligned}$$

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## 2.uzdevums

$$\begin{aligned}(q_4, 0, \square) &\rightarrow (q_F, (0, \triangle), (2, \triangle)) \\(q_4, 1, \square) &\rightarrow (q_F, (1, \triangle), (0, \triangle)) \\(q_4, 2, \square) &\rightarrow (q_F, (2, \triangle), (2, \triangle)) \\(q_4, 3, \square) &\rightarrow (q_F, (3, \triangle), (0, \triangle)) \\(q_4, 4, \square) &\rightarrow (q_F, (4, \triangle), (2, \triangle)) \\(q_4, 5, \square) &\rightarrow (q_F, (5, \triangle), (0, \triangle)) \\(q_4, 6, \square) &\rightarrow (q_F, (6, \triangle), (2, \triangle)) \\(q_4, 7, \square) &\rightarrow (q_F, (7, \triangle), (0, \triangle)) \\(q_4, 8, \square) &\rightarrow (q_F, (8, \triangle), (2, \triangle)) \\(q_4, 9, \square) &\rightarrow (q_F, (9, \triangle), (0, \triangle)) \\(q_4, \square, \square) &\rightarrow (q_F, (\square, \triangle), (2, \triangle))\end{aligned}$$

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## 2.uzdevums

$$\begin{aligned}(q_5, 0, \square) &\rightarrow (q_F, (0, \triangle), (3, \triangle)) \\(q_5, 1, \square) &\rightarrow (q_F, (1, \triangle), (1, \triangle)) \\(q_5, 2, \square) &\rightarrow (q_F, (2, \triangle), (3, \triangle)) \\(q_5, 3, \square) &\rightarrow (q_F, (3, \triangle), (1, \triangle)) \\(q_5, 4, \square) &\rightarrow (q_F, (4, \triangle), (3, \triangle)) \\(q_5, 5, \square) &\rightarrow (q_F, (5, \triangle), (1, \triangle)) \\(q_5, 6, \square) &\rightarrow (q_F, (6, \triangle), (3, \triangle)) \\(q_5, 7, \square) &\rightarrow (q_F, (7, \triangle), (1, \triangle)) \\(q_5, 8, \square) &\rightarrow (q_F, (8, \triangle), (3, \triangle)) \\(q_5, 9, \square) &\rightarrow (q_F, (9, \triangle), (1, \triangle)) \\(q_5, \square, \square) &\rightarrow (q_F, (\square, \triangle), (3, \triangle))\end{aligned}$$

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## 2.uzdevums

Ko uz 2. lentas izvadīs Tjūringa mašīna, ja uz 1. lentas ievadīts skaitlis:

- 1 20
- 2 121
- 3 2135
- 4 vispārīgā gadījumā? (Kādu algoritmu īsteno dotā Tjūringa mašīna)

## 2.uzdevums – atrisinājums

Dotā Tjūringa mašīna atlikumu, ko dod ievadītais skaitlis, dalot ar četri. Ievērojām, ka

- Pēc dalāmības pazīmes – skaitlis, dalot ar 4, dod tādu pašu atlikumu, kādu dod skaitlis, ko veido tā pēdējie divi cipari.
- Ja skaitļa pēdējie divi cipari veido skaitli  $ab$ , tad dotais skaitlis, dalot ar 4, dod tādu pašu atlikumu kā skaitlis:

$$2 \cdot a + b.$$

## 2.uzdevums – atrisinājums

Tāpēc

- ar stāvokļa  $q_0$  palīdzību tiek atrastas skaitļa beigas;
- ar stāvokli  $q_1$  tiek noteikts, kādu atlikumu dod pēdējā cipara veidotais skaitlis;
- pārejot uz stāvokļiem  $q_2, q_3, q_4, q_5$  mēs “paturam prātā” pēdējā cipara veidotā skaitļa atlikumu, dalot to ar 4 (attiecīgi – atlikums 0,1,2,3);
- Esot kādā no stāvokļiem  $q_2, q_3, q_4, q_5$  un attiecīgajā laika momentā redzot priekšpēdējo ciparu (ja tāda nav – redzam tukšumu), izvadām attiecīgo atlikumu.